TASI 2009: Supersymmetry and the MSSM

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These TASI lecture notes give an introduction to supersymmetry (SUSY) and the minimal supersymmetric standard model (MSSM). After introducing superfield and superspace formalism I will describe soft supersymmetry breaking operators, the superpartner mass spectrum, electroweak symmetry breaking, renormalisation group evolution and dark matter all within the context of the MSSM.

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1. Introduction

Although the standard model (SM) has done a remarkably good job of explaining the phenomena of the sub-atomic world there are reasons to believe that it is not the final story. In the coming years the Large Hadron Collider (LHC) will probe particle physics at unprecedented scales that, it is hoped, will reveal new laws of nature and develop the next level in our understanding of nature. One of the leading contenders for physics beyond-the-SM (BSM) is supersymmetry (SUSY), some of the other possibilities will be explained by other lecturers at TASI, and the purpose of these lectures is to explain SUSY.

In the rest of this section I will describe some problems with the SM that motivate much of BSM physics and briefly explain how SUSY deals with them. The details will occupy the rest of the notes. In Sec. 2 I will explain the modern language of SUSY, superfields and superspace, and construct simple supersymmetric Lagrangians. In Sec. 3 I will describe the field content and some features of the minimal supersymmetric version of the SM, called the MSSM, in the case where supersymmetry is unbroken. In the following section, Sec. 4, I will discuss the MSSM once SUSY is no longer an exact symmetry of the Lagrangian using the language of spurions.

One of the main motivations for introducing SUSY is problems in the Higgs sector of the SM, so in Sec. 5 I will discuss electroweak symmetry breaking in the MSSM in detail. The MSSM contains many new particles (dubbed superpartners) that can be searched for at colliders, and, once SUSY is a broken symmetry, these particles become heavy. In Sec. 6 I will calculate the mass spectra for the superpartners and touch on the issue of flavour violation generated through mixing in the superpartner sector. Discussions of SUSY often require relating physics at high scales, like the GUT scale, to physics at the weak scale. The tool for doing this is the renormalisation group, which I will discuss in Sec. 7. One of the many appealing features of SUSY is that it naturally contains within it a particle that has the properties of dark matter, and which is produced in the early universe in the correct amount. In Sec. 8 I will discuss the DM candidates within the MSSM. Finally in Sec. 9 I will not conclude, instead offering words of encouragement for your own future pursuits through the supersymmetric world!

There are many reasons to study supersymmetry, ranging from the formal to the practical; how one weighs each motivation depends on one's taste. Perhaps the best motivation to pay attention is that regardless of whether or not low energy SUSY is realised in nature it is my belief that SUSY *will* at some point be discovered at the LHC. This, seemingly rash, statement is merely a reflection of the fact that SUSY has become the benchmark for BSM physics. Despite the fact that many phenomena that are present in SUSY also present in other models of new physics, if any new physics is discovered at the LHC it will undoubtably be first attributed to some variant of SUSY. The language of supersymmetry is the *de facto* language of most collider searches for BSM physics. It is important for experimentalists and theorists alike to be well versed in the features of SUSY. Just like the ability to converse in one foreign language often aides the ability to learn another, the understanding of SUSY will aide the understanding of much of BSM physics.

The historical discovery of SUSY serves as a valuable lesson in the power of "no-go" theorems. The theorem in question is due to Coleman and Mandula[1] and, stated loosely, says that under a set of physically reasonable assumptions (*e.g.* a local, relativistic field theory) that the Lie-algebra under which the S-matrix is symmetric is at most the direct product of the Poincare group and the compact Lie group associated with internal symmetries. The major assumption, whose weakening allows for supersymmetry, is that Lie algebras are defined by commutation relations. If we allow for anti-commutation as well as commutation relations (*i.e.* the generators are no longer bosonic but may also be fermionic) we have graded Lie algebras and may avoid the Coleman Mandula theorem. This more general analysis was carried out by Haag, Lopuszanski and Sohnius[2] and they identified the most general graded Lie algebra allowed: the super-Poincare algebra.

The fact that supersymmetry is the most general space-time symmetry allowed by nature does not in principle mean it exists in nature, but it is a compelling reason to study it. SUSY involves introducing fermionic group generators, Q, and thus the action of the group, $Q|\psi\rangle = |\psi'\rangle$, must change the spin of the state. Thus, in a supersymmetric world a bosonic state has a fermionic partner and vice versa. As we will see shortly Q commutes with the Hamiltonian so these partners are degenerate in mass. Obviously this symmetry is broken in nature, what makes us believe SUSY is something we may be able to test at weak scale experiments rather than something that is broken at some high scale like the GUT scale? There are several reasons to think that SUSY may have something to do with the TeV scale and we will expound on these in more detail in these lectures.

1.1. *The Hierarchy Problem*

As is well known the Standard Model (SM) suffers from the hierarchy problem - the Higgs boson is quadratically sensitive to high scale physics. Since this is one of the main motivations for SUSY to show up at the LHC it is worth discussing the issue, and how SUSY alleviates this problem, in some detail even before we have a complete definition of what SUSY is.

The only piece of the SM not yet observed is the Higgs boson. It is also the only fundamental scalar in the theory and so behaves differently from all the other fields under quantum corrections. As a simple toy model consider a theory with a scalar (the Higgs) coupled to a heavy fermion (the top quark), for now we will ignore all gauge interactions. In the SM the fermion mass is generated from the scalar vev, here we will just insert it by hand. The Lagrangian is

$$\mathcal{L} = |\partial_\mu \phi|^2 + \overline{\psi} i \partial\!\!\!/ \psi - m_f \overline{\psi}\psi - y\phi\overline{\psi}\psi - \mu^2 |\phi|^2 - \lambda |\phi|^4 \ , \qquad (1)$$

where μ^2 is positive. Classically there is a fermion of mass m_f and a scalar of mass $m_s^2 = \mu^2$. At loop level the fermion mass term and the scalar mass term receive corrections from diagrams shown in Fig. 1. They differ in one very significant way,

$$\Delta m_f \sim -\frac{y^2}{16\pi^2} m_f \log\left(\frac{\Lambda}{m_f}\right)$$
$$\Delta\mu^2 \sim \frac{\lambda - y^2}{16\pi^2}\Lambda^2 \ . \tag{2}$$

The fermion mass corrections are *multiplicatively* renormalised whereas the scalars have an *additive* renormalisation. Thus, if the tree-level fermion masses are small they remain so after quantum corrections, whereas the scalar masses are dragged up to the cutoff scale of the theory. As expected in effective field theory (EFT), all operators allowed by symmetry are generated at the cutoff scale with $\mathcal{O}(1)$ coefficients. Here the symmetry protecting the fermion mass is a chiral symmetry, $\psi \to e^{i\alpha\gamma_5}\psi$. This is broken by the mass term and results in the loop correction being proportional to m_f. There is no such symmetry for the scalar. If the scalar were related to the fermion through a symmetry then the quadratic divergence would be removed, since it doesn't exist for the fermion. In a supersymmetric world where ψ and ϕ are related by supersymmetry we would find that λ and y are related leading to the necessary cancellation.

By supersymmetrizing the SM the quadratic divergence of the Higgs mass can be cutoff, this provides one motivation for the introduction of SUSY. The Higgs is responsible for electroweak symmetry breaking, which is associated with the $\sim 100\,\mathrm{GeV}$ scale, and in a natural theory this is the mass we would expect for the Higgs. We see from Eq. (2) that there are large quantum corrections to any bare mass the Higgs may have. If the SM is an effective theory up to high scales, for instance the GUT scale $\sim 10^{16}\,\mathrm{GeV}$, then there will be large one-loop corrections to its mass. To maintain the physical mass to be $\sim 100\,\mathrm{GeV}$ there will need to be large cancellations between the bare mass and the quantum corrections. If instead the SM becomes supersymmetric at some scale Λ_{SUSY}, *i.e.*above this scale there are superpartners of the SM fields present in the theory, these quadratic divergences will be cutoff. Requiring that there is only an $\mathcal{O}(1)$ tuning between the bare mass and the quantum corrections, cutoff at the scale Λ_{SUSY}, we expect the superpartners to enter the theory around $4\pi \times m_H \sim$ TeV.

1.2. *Dark Matter*

There is now overwhelming evidence for a large non-baryonic contribution to the matter budget of the universe. Observations over a wide range

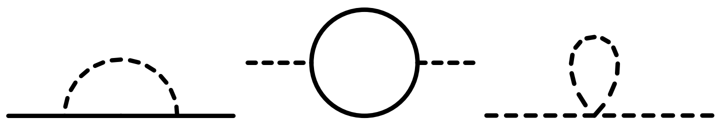

Fig. 1. One loop corrections to fermions and scalars.

of scales, such as galaxy rotation curves and measurements of the microwave background, tell us that dark matter (DM) contributes about 20% of the critical energy density of the universe. Since all observations so far[a] have been through DM's gravitational effects very little about it is known. However, we do know that it was cold, *i.e.* non-relativistic, during structure formation, it is only weakly interacting and is stable on cosmological timescales. There is no particle within the SM that satisfies these requirements so the existence of DM is clear indication of BSM physics.

Potential DM candidates include axions, black holes and weakly interacting massive particles (WIMPs) with mass ranging from $\sim$ 1GeV to $\sim 10-100$ TeV. As we will see, the MSSM contains within it a WIMP particle with the right properties to be the DM - it is absolutely stable, weakly interacting, and has mass $\sim$ 100 GeV. Even more enticing is the fact that in the thermal evolution of the universe after the big bang this particle was made in just the right abundance to explain the observed amount of DM! It seems that SUSY gives us a candidate for DM for free. It also relates what may be observed in the lab to what is being observed in the cosmos, an exciting possibility. See Sec. 8 for more details.

1.3. *Gauge coupling unification*

The gauge couplings of the SM depend on energy in a way determined by the renormalization group equations (RGEs). If one assumes that there are no new states above the weak scale, a so called desert, the three gauge couplings run in such a way that they are nearly all the same value at a high scale, $\sim 10^{14}$ GeV. This remarkable fact, that three *a priori* independent parameters have the same value at high scales is suggestive: perhaps $SU(3) \times SU(2) \times U(1)$ of the SM are really three pieces of one larger unified group, *e.g.* $SU(5)$ or $SO(10)$, that is broken at the high scale. This idea, and the models that realise it, are called GUTs, Grand Unified Theories.

[a]Recently there have been some anomalies in experiments searching directly and indirectly for dark matter that may be interpreted as observation of non-standard dark matter, see Neal Weiner's lectures Ref. 3 for more details.

However, the unification is far from perfect in the SM. Although the three lines do get close to one another at a high scale the unification is not ideal, and the scale of closest approach is low enough that proton decay, mediated by gauge bosons at the GUT scale which are left over when the GUT group is broken, should already have been observed. In the MSSM there are additional states at and just above the weak scale that will alter the RGEs and the running of the gauge couplings. Assuming that they are the only new states, *i.e.* there is a SUSY desert, one can calculate the gauge coupling running. Remarkably, the couplings now unify to a far greater degree and at a higher scale, $\sim 10^{16}$ GeV, than before, correcting both of the problems of the SM. Figure 2 shows an illustration of the the gauge coupling running, at one loop, in both the SM and the MSSM. See Sec. 7 for more discussion.

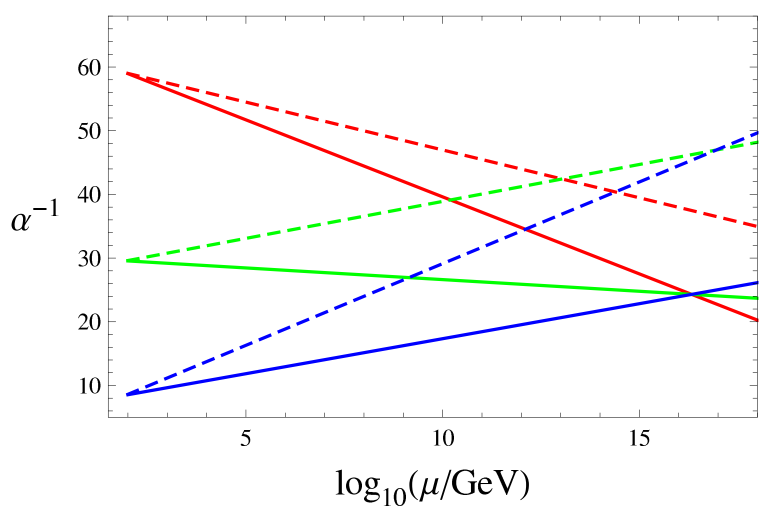

Fig. 2. One loop gauge coupling evolution for the SM (dashed lines) and the MSSM (solid lines). The $SU(3)$ gauge coupling is shown in blue (bottom lines), the $SU(2)$ in green (middle lines) and the $U(1)$, in GUT normalisation ($g_1 = \sqrt{5/3}g'$), in red (top lines).

2. Superfield (and other) formalism

"...what he needed was a notion, not a notation."
– Gauss writing about the mathematician John Wilson

In this section I will attempt to explain all the formalism necessary to understand the remainder of the lectures. Although it is not necessary to understand the superfield formalism to learn supersymmetry, it is the language used by most practitioners. It will be used by others at TASI (*e.g.* Meade and Shih) and is well worth the effort to learn. There are many other places one can look to learn the formalism, but you should be aware that they almost all use different notations and conventions, both from these lectures and each other.

I will use the "West Coast" metric, $g_{\mu\nu} = \eta_{\mu\nu} = diag(1,-1,-1,-1)$. When one first learns field theory fermions are introduced using Dirac spinors, Ψ_D. In supersymmetric field theories it is convenient to instead use Weyl spinors. For a detailed analysis of how they are related see Ref. 4. Dirac spinors are, in 4 dimensions, 4 component objects while Weyl spinors are 2 component. By working in the Weyl, or chiral, basis for the γ-matrices the relationships between the two become transparent:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \;\; \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\sigma^\mu = (1, \overrightarrow{\sigma}), \;\; \overline{\sigma}^\mu = (1, -\overrightarrow{\sigma}) \tag{4}$$

with

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; . \tag{5}$$

Recall also the combination $\sigma^{\mu\nu} = \frac{i}{4}\left(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu\right)$. The Dirac spinor may be built from a left-handed and right-handed Weyl spinor. In SUSY, and much of BSM physics, it is useful to work with only left-handed spinors. Recalling that right-handed spinors are hermitian conjugates of left-handed fields,

$$\Psi_D = \begin{pmatrix} \chi \\ \eta^\dagger \end{pmatrix} \; , \tag{6}$$

where both χ and η are left-handed. Until now I have suppressed indices, and will do so for most of the rest of the lectures, but occasional it will be necessary to include them. With indices attached Eq. (6) becomes,

$$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \eta^{\dagger\dot\alpha} \end{pmatrix} \; . \tag{7}$$

The indices are raised and lowered with $\epsilon_{\alpha\beta}$ and $\epsilon^{\alpha\beta}$ with $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1$, all others 0. Spinor summations are defined as

$$\chi\eta \equiv \chi^\alpha\eta_\alpha \, , \, \chi^\dagger\eta^\dagger \equiv \chi^\dagger_{\dot\alpha}\eta^{\dagger\dot\alpha} \; . \tag{8}$$

Once these spinor summation conventions are defined we can usually get away with suppressing the indices.

Exercise: Show $\chi\eta = \eta\chi$.

2.1. *Superspace*

With the addition of supersymmetry the usual algebra of the Lorentz group is extended by the supersymmetry algebra which, for $N = 1$ supersymmetry in 4 dimensions, is

$$\left\{Q_\alpha, Q^\dagger_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu \tag{9}$$

$$\{Q_\alpha, Q_\beta\} = \left\{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\right\} = 0 \tag{10}$$

$$[P_\mu, Q_\alpha] = \left[P_\mu, Q^\dagger_{\dot\alpha}\right] = 0 \tag{11}$$

The generators of the SUSY algebra, Q_α are spinors and SUSY transformations are of the form boson $\leftrightarrow$ fermion. Equation (11) indicates that SUSY transformations commute with the Hamiltonian and states related by a SUSY transformation have the same mass, such states are called superpartners. From Eq. (9) we see that two SUSY transformations amount to a spacetime translation *i.e.* supersymmetry is a spacetime symmetry. This suggests the concept of superspace, augmenting the usual four (commuting) coordinates x^μ to include 4 anticommuting (Grassmann) coordinates θ_α, $\overline{\theta}_{\dot\alpha} \equiv (\theta_\alpha)^\dagger$. Recall the features of Grassmann spinors:

$$\left\{\theta^\alpha, \theta^\beta\right\} = \left\{\overline{\theta}_{\dot\alpha}, \overline{\theta}_{\dot\beta}\right\} = \left\{\theta^\alpha, \overline{\theta}_{\dot\beta}\right\} = 0\ , \tag{12}$$

leading to the result that the square of a Grassmann coordinate is zero, making for simple Taylor series. For Grassmann variables integration is akin to differentiation and,

$$\int d^2\theta\, \theta^2 \equiv \int d^2\theta\theta^\alpha\theta_\alpha = 1 \quad \int d^2\theta d^2\overline{\theta}\, \theta^2\overline{\theta}^2 = 1 \tag{13}$$

Exercise: Show $d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta\epsilon_{\alpha\beta}$ and $\frac{\partial^2}{\partial\theta^\alpha\partial\theta_\alpha}\theta^2 = 4$.

Just as the momentum operator, $-i\partial_\mu$, is the generator of space-time translations we would like to determine the generator, Q_α, of SUSY transformations. An obvious guess is $Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha}$ but it is easy to check that this does not satisfy the algebra of Eq. (9). Instead the generators are,

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\mu \tag{14}$$

$$\overline{Q}_{\dot\alpha} = \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \tag{15}$$

Exercise: Show that these Q do indeed satisfy the SUSY algebra.

With the generators in hand we may exponentiate and carry out a finite SUSY transformation on a function of superspace, which has a remarkably simple form.

Exercise: Confirm that

$$e^{\epsilon Q+\overline{\epsilon}\overline{Q}}f(x^\mu,\theta,\overline{\theta}) = f(x^\mu + i\epsilon\sigma^\mu\overline{\theta} + i\theta\sigma^\mu\overline{\epsilon}, \theta+\epsilon, \overline{\theta}+\overline{\epsilon})\ . \tag{16}$$

The final piece we need to introduce are the superspace derivatives, which anti-commute with the generators and are given by[b]

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\left(\sigma^\mu\overline{\theta}\right)_\alpha\partial_\mu \tag{17}$$

$$\overline{D}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i\left(\theta\sigma^\mu\right)_{\dot\alpha}\partial_\mu \tag{18}$$

$$\tag{19}$$

So far this may seem like formality for formality's sake, but its utility will hopefully soon become very clear. Rather than working with component fields, *e.g.* fermions and scalars, and constructing Lagrangians that must be painstakingly checked to ensure SUSY is preserved we can instead work with superfields and supersymmetry is ensured. It is much like using four vectors in relativity, if there are no "hanging indices" then Lorentz invariance is maintained without having to worry about how $t, x, y,$ and z transform under a particular boost. In addition, actions are now built from integrals over superspace, $\int d^4x d^2\theta d^2\overline{\theta}$.

Thanks to the properties of Grassmann coordinates Eq. (12) the most general superfield can be Taylor expanded in its θ coordinates.

$$G(x,\theta,\overline{\theta}) = \phi(x)+\theta\psi+\overline{\theta}\overline{\chi}+\theta^2 m+\overline{\theta}^2 n+\theta\sigma^\mu\overline{\theta}V_\mu+\theta^2\overline{\theta}\overline{\lambda}+\overline{\theta}^2\theta\rho+\theta^2\overline{\theta}^2 d \tag{20}$$

[b]Notice that I have (deliberately) started to become more sloppy with indices, but there is still enough information to replace them all, should you feel so inclined.

Table 1. Number of degrees of freedom of components of the chiral multiplet.

Field	Off-shell	On-shell
ϕ	2	2
ψ	4	2
F	2	0

This is a *lot* of fields, more than we would expect to realise supersymmetry given the toy example discussed in the introduction. This general representation Eq. (20) is reducible, and by imposing constraints we can build smaller irreducible representations. It is these we will use to describe supersymmetric field theories.

2.2. *Chiral Superfield*

We can build a smaller representation, the chiral superfield, by imposing the constraint

$$\overline{D}\Phi = 0 \; . \tag{21}$$

Notice that since $\{D, Q\} = 0$ this constraint is invariant under SUSY transformations. To identify what a chiral superfield is in terms of components first note that

$$\overline{D}_{\dot{\alpha}} \left(x^\mu + i\theta\sigma^\mu\overline{\theta} \right) = 0 \quad \text{and} \quad \overline{D}_{\dot{\alpha}}\theta = 0 \; . \tag{22}$$

Thus, a chiral superfield is a function of $y = x^\mu + i\theta\sigma^\mu\overline{\theta}$ and θ. Then, expanding as before in powers of θ,

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{23}$$

$$\begin{aligned} &= \phi(x) - i\theta\sigma^\mu\overline{\theta}\partial_\mu\phi - \frac{1}{4}\theta^2\overline{\theta}^2\partial^2\phi \\ &\quad + \sqrt{2}\theta\psi + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\overline{\theta} + \theta^2 F \; . \end{aligned} \tag{24}$$

So we see that the chiral superfield contains a complex scalar, ϕ, a Weyl fermion, ψ and another complex scalar, F, that we will refer to as an auxiliary field (we will see why shortly). It is the perfect candidate to use for the matter and Higgs fields in a supersymmetric version of the SM. Note also that any analytic function of chiral superfields (*i.e.* a function made out of powers of Φ and no powers of $\Phi^\dagger$) is itself a chiral superfield.

Exercise: Using the results of the previous exercise work out the SUSY transformations on the components of the chiral superfield. That is, calculate $\delta\Phi = \left(\epsilon Q + \overline{\epsilon}\overline{Q}\right)\Phi$ and confirm that,

$$\delta\phi = \sqrt{2}\epsilon\psi, \quad \delta\psi = \sqrt{2}\epsilon F + \sqrt{2} i\sigma^\mu\overline{\epsilon}\partial_\mu\phi, \quad \delta F = i\sqrt{2}\overline{\epsilon}\overline{\sigma}^\mu\partial_\mu\psi \ . \tag{25}$$

Chiral superfields can be combined in various ways to build superspace, and therefore supersymmetric, invariants. From Eq. (21) we see that any holomorphic function of chiral superfields is itself a chiral superfield. Also, notice that the highest component of the chiral superfield transforms into a total derivative under a SUSY transformation (see the previous exercise Eq. (25)). This is true for the highest component of any supermultiplet and is as expected on dimensional grounds; since F is the highest dimension field in the multiplet and the SUSY transformation involves ϵ whose dimension is $[\epsilon] = -1/2$, making up the units requires a derivative. Since any holomorphic function of chiral superfields is itself a chiral superfield, then the quantity

$$\int d^4x \int d^2\theta\, W(\Phi) \ , \tag{26}$$

where W is a polynomial in Φ, is a SUSY invariant and a perfect candidate for a term in a SUSY action. Thus, for chiral superfields an integral over half of superspace is invariant. Alternatively, $\overline{\theta}^2 f(\Phi)$ is invariant when integrated over all of superspace but using Eq. (13) this reduces to integrating over only θ^2.

Functions of both Φ and $\Phi^\dagger$ must be integrated over the whole of superspace in order to be invariant. Thus, we can now write down the most general supersymmetric invariant action built from chiral superfields, Φ_i,

$$S = \int d^4x \left[\int d^4\theta\, K(\Phi_i^\dagger, \Phi_j) + \int d^2\theta\, W(\Phi_i) + h.c.\right] \ . \tag{27}$$

K is the Kähler potential and is real and W is the superpotential and is holomorphic in the chiral superfield(s). The chiral superfield has dimension $[\Phi] = 1$, the same as for its scalar ϕ, which means that $[\theta] = -1/2$. So the Kähler potential must have dimension 2 and the superpotential dimension 3, which will limit the renormalizable terms we can write down. Let us examine a simple example of a supersymmetric theory constructed entirely from chiral superfields. In so doing some of the formalism's utility will become apparent.

2.2.1. *Wess-Zumino model*

The most general supersymmetric, renormalizable model of a single chiral superfield has Lagrangian density

$$\int d^4\theta\, \Phi^\dagger \Phi + \int d^2\theta \left(\frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3 \right) + h.c. \tag{28}$$

Using the results of the previous subsection we can expand the superfield in its components and find

$$\begin{aligned} \mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi + \psi^\dagger i\overline{\sigma}^\mu \partial_\mu \psi + F^* F \\ + mF\phi - \frac{1}{2} m\psi\psi + h.c. + \lambda F \phi^2 - \lambda \phi\psi\psi + h.c. \end{aligned} \tag{29}$$

The first line comes from the Kähler potential in Eq. (28) and the second from the superpotential. This looks like a model of an interacting Weyl fermion and a complex scalar very similar to that discussed in the introduction, but what about F? There is no $\partial F/\partial t$ term in the Lagrangian. It is not a propagating field so its equations of motion will be algebraic, hence the name auxiliary field. This explains the counting shown in Table 1, after application of the equations of motion the only degrees of freedom are contained in the fermion and boson and match. But off-shell, where the equations of motion are not applied, we need to introduce additional bosonic degrees of freedom. The introduction of the auxiliary fields and of the superspace notation gives a representation of supersymmetry that closes even off-shell.

Since the F-term equations are algebraic in the other fields they can be solved for and re-inserted into the Lagrangian. For the simple case with canonical Kähler potential, $K = \Phi^\dagger\Phi$, the F-term equations of motion are

$$F^* = -\frac{\partial W}{\partial \phi} \; . \tag{30}$$

Inserting these equations back into the action results in a contribution to the potential from these F-terms,

$$V_F = |F|^2 = |\frac{\partial W}{\partial \phi}|^2 \; , \tag{31}$$

notice that this potential is positive semi-definite.

Doing this for the Wess-Zumino model we find

$$F^* = -\frac{\partial W}{\partial \phi} = -(m\phi + \lambda\phi^2) \; , \tag{32}$$

and then

$$\mathcal{L} = |\partial_\mu \phi|^2 + \psi^\dagger i\overline{\sigma}^\mu \partial_\mu \psi - \frac{1}{2} m\psi\psi - \lambda\phi\psi\psi + h.c. - |m\phi + \lambda\phi^2|^2 \ . \qquad (33)$$

This is then a model of a fermion interacting with a scalar. They are degenerate in mass, and if you were to calculate the loop corrections to the scalar masses you would find there is no quadratic divergence. This last statement is easy to see from the example in Sec. 1.1, supersymmetry relates the Yukawa coupling to the scalar self coupling and the quadratic divergence of Eq. (2) is cancelled. The additional scalar^3 coupling present in the Wess-Zumino model cannot introduce quadratic divergences in the scalar mass^2 since the coupling is dimensionful. Furthermore, because the scalar and fermion masses are the same all logarithmic divergences also cancel.

For completeness, the general case, with arbitrary number of chiral superfields Φ_i, where the Lagrangian is given beyEq. (27) leads to a potential

$$V = \frac{\partial W^*}{\partial \phi_i^*} K_{ij}^{-1} \frac{\partial W}{\partial \phi_j}, \text{ where } K_{ij} = \frac{\partial K}{\partial \phi_i^* \partial \phi_j} \ . \qquad (34)$$

2.3. *Vector Superfield*

Another constraint that can be placed on the general superfield is that of reality,

$$V^\dagger = V \ . \qquad (35)$$

Doing so will lead us to the vector superfield. The full vector superfield still has many components but we can take advantage of the fact that all the vectors in the SM are gauge bosons and have a related gauge symmetry[c], $A_\mu \to A_\mu + \partial_\mu \Lambda$, to try to gauge some of the components away. We extend the gauge transformations to act on superfields by noticing that for a chiral superfield Λ the combination $\Lambda + \Lambda^\dagger$ is real so $V + (\Lambda + \Lambda^\dagger)$ is still a vector superfield. In addition, both expansions contain terms that behave in the correct way to be the symmetry transformation on the gauge field,

$$V = \ldots + \theta\sigma^\mu\overline{\theta} A_\mu + \ldots, \quad \text{and} \quad \Lambda + \Lambda^\dagger = \ldots + i\theta\sigma^\mu\overline{\theta}\partial_\mu(\phi - \phi^\dagger) + \ldots \quad (36)$$

Using this gauge transformation we can write the vector superfield in the Wess-Zumino gauge where many of the components have been gauged away, leaving just a vector, a fermion and a real scalar (another auxiliary field),

$$V \overset{WZ gauge}{=} 2\theta\sigma^\mu\overline{\theta} A_\mu + 2\theta^2\overline{\theta}\lambda^\dagger + 2\theta\overline{\theta}^2\lambda + \theta^2\overline{\theta}^2 D \ . \qquad (37)$$

[c]For now we restrict ourselves to Abelian groups.

Thus the vector multiplet contains the gauge fields and their superpartners, the gauginos. In order to write down the kinetic terms for the gauge fields and its superpartner we introduce the (gauge covariant) chiral superfield, W_α, built from the vector superfield,

$$W_\alpha = -\frac{1}{8}\overline{D}^2 D_\alpha V, \text{ and } \overline{W}_{\dot\alpha} = -\frac{1}{8} D^2 \overline{D}_\alpha V \tag{38}$$

Expanding in terms of component fields leads to,

$$W_\alpha = \lambda_\alpha + \theta_\alpha D - (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu} + i\theta^2 \sigma^\mu \partial_\mu \lambda^\dagger \ , \tag{39}$$

and explains the often used name of supersymmetric field strength. The field strength has scaling dimension $[W_\alpha] = 3/2$ and the only renormalizable operator we can build from it is a superpotential term,

$$\begin{aligned} &\frac{1}{8\pi}\mathrm{Im}\left[\left(\frac{4\pi i}{g^2} + \frac{\theta_{YM}}{2\pi}\right)\int d^2\theta W^\alpha W_\alpha\right] \\ &= -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{g^2}\lambda^\dagger \overline{\sigma}^\mu D_\mu \lambda + \frac{1}{2g^2}D^2 - \frac{\theta_{YM}}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \ . \end{aligned} \tag{40}$$

Exercise: By applying the SUSY generators (15) to (37) show that restricting to the Wess-Zumino gauge breaks supersymmetry.

It is common to treat the combination of gauge coupling and θ-angle as one quantity, a complex gauge coupling, $\tau = \frac{4\pi i}{g^2} + \frac{\theta_{YM}}{2\pi}$, and for most discussions it is sufficient to assume $\theta_{YM} = 0$. In this case the Lagrangian term is,

$$\frac{1}{4g^2}\int d^2\theta \, W^\alpha W_\alpha + h.c. = -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{g^2}\lambda^\dagger \overline{\sigma}^\mu D_\mu \lambda + \frac{1}{2g^2}D^2 \ . \tag{41}$$

As expected the auxiliary field, D, has no kinetic term and again its equation of motion will be algebraic.

If the chiral superfields of the previous section are charged under the gauge group then they transform as,

$$\Phi \to e^{-q\Lambda}\Phi \ , \tag{42}$$

which means that the Kähler potential of Eq. (27) is no longer gauge invariant. Including the gauge interactions the most general Lagrangian involving vector and chiral superfields of charge q_i becomes,

$$\mathcal{L} = \int d^4\theta K\left(\Phi_i^\dagger, e^{q_i V}\Phi_i\right) + \int d^2\theta \, \tau \, W_\alpha W^\alpha + h.c. + \int d^2\theta W(\Phi_i) + h.c. \tag{43}$$

Table 2. Number of degrees of freedom of components of the vector multiplet.

Field	Off-shell	On-shell
A_μ	3	2
λ	4	2
D	1	0

We will limit ourselves to the canonical (renormalizable) Kähler term, $K = \Phi^\dagger e^{qV}\Phi$ for which,

$$\int d^4\theta\, \Phi_i^\dagger e^{q_i V}\Phi_i = D^\mu \phi_i^* D_\mu \phi_i + \psi_i^\dagger i\overline{\sigma}^\mu \partial_\mu \psi_i + F_i^* F_i \\ +\sqrt{2}\sum_i q_i \left(\phi_i^* \psi_i \lambda + \lambda^\dagger \psi_i^\dagger \phi_i\right) + \sum_i q_i D \phi_i^* \phi_i \quad (44)$$

Combining this with Eq. (40) we can solve for the D-term and find,

$$D = -g^2 \sum_i q_i \phi_i^* \phi_i \ . \quad (45)$$

As before we can remove the auxiliary field from the Lagrangian and we find that it contributes to the potential,

$$V_D = \frac{1}{2} g^2 \left(\sum_i q_i \phi_i^* \phi_i\right)^2 \ . \quad (46)$$

So far we have limited ourselves to Abelian groups. For non-Abelian groups chiral multiplets whose representation have generators T^a, transform as,

$$\Phi \to e^{-T^a\Lambda^a}\Phi,\ \Phi^\dagger \to \Phi^\dagger e^{-T^a\Lambda^{a\dagger}} \quad (47)$$

in particular fundamental and anti-fundamental representations have a relative minus sign in the way they transform. The vector superfield now has a more complicated transformation,

$$e^{T^a V^a} \to e^{T^a\Lambda^{a\dagger}} e^{T^a V^a} e^{T^a \Lambda^a} \quad (48)$$

and the supersymmetric field strength is now,

$$W_\alpha^a T^a = -\frac{1}{4}\overline{D}^2 e^{-T^a\Lambda^a} D_\alpha e^{T^a V^a} \ . \quad (49)$$

For the particular case of an Abelian group there is one more supersymmetric and gauge invariant term we can add to the Lagrangian, the Fayet-Iliopolis term,

$$\xi \int d^4\theta V = \xi D \ , \quad (50)$$

which acts as a source for the D-term.

In a general theory involving chiral and vector superfields the scalar potential is given by the sum of F-term and D-term contributions,

$$V = V_F + V_D \ , \tag{51}$$

and it is positive semi-definite, $V \geq 0$. In fact, if and only if the F-term and D-term equations can be solved[d] (*i.e.* $F_i = 0$ and $D^a = 0$) is supersymmetry unbroken. To see this recall the susy algebra Eq. (9) and take the expectation value of the trace of Eq. (9) in the vacuum,

$$\begin{aligned} \langle 0|4P^0|0\rangle &= \langle 0|\{Q_\alpha, Q^\dagger_{\dot\alpha}\}|0\rangle = \langle 0|(Q_1Q_1^\dagger + Q_1^\dagger Q_1 + Q_2Q_2^\dagger + Q_2^\dagger Q_2)|0\rangle \\ &= \left|Q_1^\dagger|0\rangle\right|^2 + |Q_1|0\rangle|^2 + \left|Q_2^\dagger|0\rangle\right|^2 + |Q_2|0\rangle|^2 \\ &\geq 0 \end{aligned} \tag{52}$$

If the vacuum $|0\rangle$ is invariant under a supersymmetric transformation then $Q|0\rangle = 0$ and SUSY is unbroken and the vacuum energy $\langle 0|H|0\rangle = 0$ and thus $F = 0$ and $D = 0$. Otherwise if SUSY is spontaneously broken (Q does not annihilate the vacuum) the vacuum energy is positive, since the right side of Eq. (52) is positive semi-definite, and one of the F or D-terms is non-zero.

2.4. *R-symmetry*

With the introduction of superspace coordinates it is possible to define a new symmetry of the action. Under this R-symmetry the θ coordinate picks up a phase,

$$\theta \to e^{i\alpha}\theta, \quad \text{and} \quad \overline{\theta} \to e^{-i\alpha}\overline{\theta} \tag{53}$$

From our definition of integration of Grassmann coordinates Eq. (13) we see $d\theta$ rotates the opposite way to θ. This means that if the Kähler potential has R-charge 0 and the superpotential has R-charge 2 the action will be R-symmetric. One immediate consequence of this is that W_α and therefore gauginos have R-charge 1. Under an R-symmetry transformation θ rotates by a phase, so different components of a superfield must have different R-charges. As an example consider the superpotential $W = m\Phi^2$ which is R-symmetric if Φ has R-charge 1, its components then transform as,

$$\phi(x) \to e^{i\alpha}\phi(x),\ \psi(x) \to \psi(x),\ F \to e^{-i\alpha}F \ . \tag{54}$$

[d]In non-Abelian theories the existence of a supersymmetric vacuum is determined entirely by the F-term equations.

2.5. *Putting the formalism to work: O'Raifeartaigh and other models*

So far we have concentrated on writing down supersymmetric actions without worry about whether the ground state is supersymmetric. Now we will consider the simplest class of models that spontaneously break SUSY, and in so doing learn a few general rules about models that break SUSY at tree level and how one goes about analyzing models of SUSY breaking.

The simplest models[e] that break supersymmetry are O'Raifeartaigh models[6] and are built from chiral superfields. Consider as an example the model with 3 chiral superfields, A, B, X and superpotential,

$$W = \lambda X(A^2 - \mu^2) + mAB + h.c. \ , \tag{55}$$

we will assume throughout that the parameters are all real. The F-term equations are,

$$F_X^* = -\frac{\partial W}{\partial X} = \lambda(A^2 - \mu^2) = 0 \tag{56}$$

$$F_A^* = -\frac{\partial W}{\partial A} = mB + 2\lambda AX = 0 \tag{57}$$

$$F_B^* = -\frac{\partial W}{\partial B} = mA = 0 \ , \tag{58}$$

which cannot be simultaneously solved and thus SUSY is broken. It is instructive to examine the spectrum in this model, to do so we will need the fermion, M_F, and scalar, M_S^2, mass matrices. At tree-level these are simply given by,

$$M_F|_{ij} = \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} \quad \text{and} \quad M_S^2|_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \tag{59}$$

For the O'Raifeartaigh model of interest the potential is given by,

$$V = |F_X|^2 + |F_A|^2 + |F_B|^2 = |\lambda(A^2 - \mu^2)|^2 + |mB + 2\lambda AX|^2 + |mA|^2 \ , \tag{60}$$

which has a flat direction since Eq. (57) can always be solved regardless of the values of the other fields. This vacuum degeneracy will be lifted by loop corrections. If $m^2 - 2\lambda^2\mu^2 > 0$ the minimum is at the origin, otherwise A acquires a vev. The two minima are

$$A = 0, B = 0 \tag{61}$$

$$A^2 = \frac{2\lambda^2\mu^2 - m^2}{2\lambda^2}, B = \frac{2\lambda}{m}\sqrt{\frac{2\lambda^2\mu^2 - m^2}{2\lambda^2}}X. \tag{62}$$

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[e]The Poloyni model[5] has just a linear superpotential, $W = \mu^2 Z$, and is simpler, but rather boring to analyze.

At the first, $V = \lambda^2\mu^4$ and at the second $V = m^2(\mu^2 - \frac{m^2}{4\lambda^2})$. Concentrating on the case with the vacuum at the origin the fermion mass matrix in the (ψ_X, ψ_A, ψ_B) basis is given by,

$$M_F = \frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\lambda x & m \\ 0 & m & 0 \end{pmatrix} , \tag{63}$$

where $x = \langle X\rangle$. The three fermions have mass 0, and $\lambda x \pm \sqrt{m^2 + \lambda^2\lambda x^2}$. The massless fermion is the *Goldstino*, the analogue of the Goldstone boson of spontaneously broken global symmetries. Here it is fermionic since the spontaneoulsy broken symmetry is SUSY and its generators are fermionic not bosonic.

The scalar mass matrix is more complicated. In principle it is a 6×6 matrix but since X and X^* don't acquire masses we concentrate on the 4×4 submatrix. In the (A, B, A^*, B^*) basis it is,

$$M_S^2 = \frac{\partial^2 V}{\partial\phi_i\partial\phi_j} = \begin{pmatrix} m^2 + 4\lambda^2x^2 & 2\lambda mx & -\lambda^2\mu^2 & 0 \\ 2\lambda mx & m^2 & 0 & 0 \\ -\lambda^2\mu^2 & 0 & m^2 + 4\lambda^2x^2 & 2\lambda mx \\ 0 & 0 & 2\lambda mx & m^2 \end{pmatrix} \tag{64}$$

The scalar masses (really m^2's) are 0,0, and $m^2 + \frac{\lambda}{2}(4\lambda x^2 \pm \lambda\mu^2 \pm \sqrt{16m^2x^2 + \lambda^2(\mu^2 - 4x^2)})$. We can immediately see another feature of spontaneous SUSY breaking in a renormalizable theory, there is a sum rule:

$$\text{Str } M^2 = \sum(-1)^{2J}(2J+1)M_J^2 = \sum_{scalars} M_s^2 - 2\sum_{fermions} M_F^2 = 0 \tag{65}$$

This is true in all theories where SUSY is broken at the renormalizable level and immediately indicates a problem for coupling the MSSM to SUSY breaking directly - there would be superpartner lighter than its SM partner!

To see that this is true in general and not just a quirk of O'Raifeartaigh models recall that the scalar mass matrix is of the form,

$$M_S^2 : \frac{1}{2}\left(\Phi_i^*\Phi_i\right)\begin{pmatrix} \frac{\partial^2 V}{\partial\phi_i^*\partial\phi_j} & \frac{\partial^2 V}{\partial\phi_i^*\partial\phi_j^*} \\ \frac{\partial^2 V}{\partial\phi_i\partial\phi_j} & \frac{\partial^2 V}{\partial\phi_i\partial\phi_j^*} \end{pmatrix}\begin{pmatrix} \Phi_j \\ \Phi_j^* \end{pmatrix} \tag{66}$$

while the fermion mass matrix is,

$$M_F : \frac{1}{2}\psi_i\frac{\partial W}{\partial\phi_i\partial\phi_j}\psi_j \tag{67}$$

Since $V = \frac{\partial W}{\partial \phi_i}\frac{\partial W^*}{\partial \phi_i^*}$ we immediately see that $TrM_S^2 = 2TrM_F^2$.

Exercise: Fayet-Iliopoulos terms. For a $U(1)$ gauge group there is one more gauge invariant operator that can be added to the Lagrangian, a Fayet-Iliopoulos term, $\int d^4\theta\kappa V$. Consider SUSY QED with an FI term and a vector like pair of "electrons", *i.e.*

$$\left(\Phi_1^\dagger e^{eV}\Phi_1 + \Phi_2^\dagger e^{-eV}\Phi_2 - \kappa^2 V\right)\Big|_{\theta^4} + \left(\frac{1}{4}W_\alpha W^\alpha + m\Phi_1\Phi_2 + h.c.\right)\Big|_{\theta^2} \tag{68}$$

Show that for the case $m^2 > e\kappa^2$ SUSY is broken but the gauge symmetry is not but for $m^2 < e\kappa^2$ both SUSY and the $U(1)$ are broken. Show that in both cases the supertrace is 0, as expected.

3. The MSSM - unbroken SUSY

Now we are in a position to discuss the supersymmetric version of the SM. There are many ways in which one can imagine embedding the SM within supersymmetry, the one which requires the introduction of the smallest number of superpartners is called the Minimal Supersymmetric Standard Model (MSSM). Before writing down its Lagrangian it is useful to first remind ourselves of the field content of the SM, written in terms of only LH fermions. The SM is based on the gauge structure $SU(3)\times SU(2)\times U(1)$ and under these groups it has 3 generations of matter fields that are in the following representations:

$$\begin{aligned} q_i = (u_L, d_L) &: \left(3, 2, \frac{1}{6}\right) \\ u_i^c &: \left(\overline{3}, 1, -\frac{2}{3}\right) \\ d_i^c &: \left(\overline{3}, 1, \frac{1}{3}\right) \\ \ell = (\nu, e_L) &: \left(1, 2, -\frac{1}{2}\right) \\ e_i^c &: (1, 1, 1) \end{aligned} \tag{69}$$

Gauge fields that are in the adjoint representation of the groups:

$$\begin{aligned} g &: (8, 1, 0) \\ A_\mu^a &: (1, 3, 0) \\ B_\mu &: (1, 1, 0) \end{aligned} \tag{70}$$

The last two mix after electroweak symmetry breaking. Finally there is the Higgs boson:

$$h : \left(1, 2, \frac{1}{2}\right) \tag{71}$$

The simplest way to supersymmetrise is to place all the SM fields into superfields and introduce the necessary superpartners to fill out the superfields. For the fermions this requires introducing scalars (dubbed sfermions) and placing them in a chiral multiplets. We will denote the chiral superfield by the upper case version of the SM field, *e.g.* $q_i \to Q_i$. Superpartners of SM fields will be denoted with a tilde and the scalars have been given names by (unfortunately) adding an "s" to the front of the SM particles name, *e.g.* the superpartner of the electron (the selectron) is $\tilde{e}$.

The gauge bosons will require the introduction of femionic partners (dubbed gauginos) and will be placed in vector superfields. We will denote them as V_i where $i = 3, 2, 1$ denotes the rank of the group. The fermionic partners take their name from the SM field and adding an "ino" on the end, *e.g.* the gluino, $\tilde{g}$, is the fermionic partner of the gluon, g.

So far in filling out the chiral superfields we have been introducing new bosonic partners. In the case of the Higgs however we are introducing a new chiral *fermion* and this leads to a problem. Chiral fermions contribute to anomalies and the introduction of one fermion charged under $SU(2) \times U(1)$ will make the gauge symmetries anomalous. Also, the restriction in supersymmetry that the superpotential has to be a holomorphic function of the chiral superfields would forbid some of the necessary Yukawa couplings. Both of these facts can be avoided if we introduce not only a fermionic partner of the SM Higgs (by the naming convention called a Higgsino) but a second chiral superfield. Thus there are now two Higgs chiral superfields,

$$\begin{aligned} H_u &= (H_u^+, H_u^0) : \left(1, 2, \frac{1}{2}\right) \\ H_d &= (H_d^0, H_d^-) : \left(1, 2, -\frac{1}{2}\right) \end{aligned} \tag{72}$$

The total field content and the bizarre naming convention is collected in Table 3.

With the field content in hand we may now procede to follow the mantra of effective field theory and write down all operators allowed by symmetry. Keeping only renormalizable operators we have Kähler terms of the form,

$$K = Q^\dagger e^{V_3} Q + U^{c\dagger} e^{-V_3} U^c + D^{c\dagger} e^{-V_3} D^c + \ldots \tag{73}$$

Table 3. Field content and naming conventions of the MSSM.

SM Field	$SU(3), SU(2), U(1)$	MSSM partner	Superfield
q_i (LH quarks)	$(3, 2, \frac{1}{6})$	$\tilde{q}_i$ (LH squarks)	Q_i
u_i^c (RH top, charm, up)	$(\bar{3}, 1, -\frac{2}{3})$	$\tilde{u}_i^c$ (RH stop, scharm, sup)	U_i^c
d_i^c (RH bottom, strange, down)	$(\bar{3}, 1, \frac{1}{3})$	$\tilde{d}_i^c$ (RH sbottom, sstrange, sdown)	D_i^c
ℓ_i (LH leptons)	$(1, 2, -\frac{1}{2})$	$\tilde{\ell}_i$ (LH sleptons)	L_i
e_i^c (RH tau, muon, electron)	$(1, 1, 1)$	$\tilde{e}_i^c$ (RH stau, smuon, selectron)	E_i^c
h_u (h_d) (Higgs)	$(1, 2, \frac{1}{2})$; $(1, 2, -\frac{1}{2})$	$\tilde{h}_u$ $\left(\tilde{h}_d\right)$ (higgsino)	H_u (H_d)

gauge kinetic terms of the form,

$$\int d^2\theta \, \frac{1}{4g_3^2} W_\alpha^{(3)} W^{(3)\alpha} + \ldots \; . \tag{74}$$

Finally, the superpotential which we discuss in two parts. First,

$$W_{MSSM} = \mathbf{Y_U} U^c Q H_u - \mathbf{Y_D} D^c Q H_d - \mathbf{Y_E} E^c L H_d + \mu H_u H_d \; . \tag{75}$$

I have suppressed flavour and gauge indices for clarity. We can see again the need for the introduction of a second Higgs doublet, without it some of the SM fermions would be massless. As in the SM the fields may be rotated such that the Yukawas are diagonal, and since the third generation of SM fermions is appreciably heavier than first two the Yukawas are often approximated as $\mathbf{Y_U} \approx diag(0, 0, y_t)$, $\mathbf{Y_D} \approx diag(0, 0, y_b)$ and $\mathbf{Y_E} \approx diag(0, 0, y_\tau)$. The μ-term is a mass term for the Higgsinos and will also, through F-terms, contribute to the scalar potential.

Expanding the superfields in W_{MSSM} in their component fields gives us the Feynman rules for the SM particles and their superpartners. Concentrating on the top Yukawa term we can write down three different couplings all of size y_t and we learn a very useful rule of thumb for understanding couplings in the MSSM, see Fig. 3. Take any vertex in the SM and replace two of the particles with their superpartners and this is a vertex in the MSSM. This does not capture all the available couplings, for instance the F-term for U^c leads to a four-point Higgs-squark coupling that has no SM counterpart, but does work for couplings involving at least one SM fermion coming from the superpotential and the gauge coupling terms.

Exercise: Put the flavour and gauge indices back into Eq. (75), paying close attention to $SU(2)$ indices which are contracted with $\epsilon_{\alpha\beta}$, and confirm the signs.

In addition to these SM-like terms there are some other renormalisable operators allowed by the gauge symmetries that can be added to the

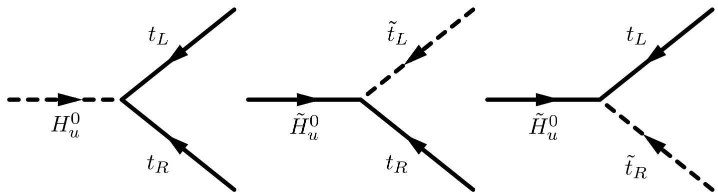

Fig. 3. Top Yukawa couplings.

superpotential,

$$W_{\Delta B,L} = \kappa_1^{ijk} Q_i L_j D_k^c + \kappa_2^{ijk} L_i L_j E_k^c + \kappa_3^i L^i H_u + \kappa_4^{ijk} D_i^c D_j^c U_k^c \ . \qquad (76)$$

However, the first 3 of these operators violate lepton number, and the last is no better since it violates baryon number. Note that both κ_2 and κ_4 are antisymmetric under $i \leftrightarrow j$ because of the antisymmetry of the gauge indices, one is contracted with $\epsilon^{\alpha\beta}$ and one with f^{abc}. At the renormalizable level in the SM baryon and lepton symmetries are accidental, operators that would violate B or L are forbidden because of gauge symmetries; B and L are separately violated by non-perturbative processes, only $B - L$ is conserved. In the MSSM this accident no longer happens because superpartners allow us to construct the operators in Eq. (76). We could forbid these operators by fiat, as we will see the superpotential has an interesting non-renormalisation property so that even if there is no symmetry forbidding these operators once their coefficients are set to zero they won't be generated in perturbation theory, but this is not appealing.

These operators could be forbidden if we introduced a new symmetry, the price we have to pay for wanting to solve the hierarchy problem. For instance we could introduce an R-symmetry as inSec. 2.4 where $R[Q, U^c, D^c, L, E^c] = 1/2$ and $R[H_u, H_d] = 1$. This would forbid the $W_{\Delta B,L}$ terms while allowing the W_{MSSM} terms. However, as we will soon see, this is too restrictive and would forbid mass terms for gauginos. Instead we consider a discrete Z_2 subgroup of the $U(1)$ R-symmetry under which superpartners flip sign and SM fields do not. Under this R-parity[f] the fields have charge,

$$P_R = (-1)^{3(B-L)+F} \qquad (77)$$

Under the parity SM fields are even and superpartners are odd and it has several interesting implications:

[f]Equivalently another possibility is matter parity where parity is assigned by $P_M = (-1)^{3(B-L)}$.

(1) Superpartners and SM particles cannot mix
(2) The lightest parity odd particle (LPOP) is a superpartner and the lightest supersymmetric particle (LSP) is stable. It turns out that the LSP is often a neutral state and has exactly the right properties to be the DM!
(3) Superpartners must be made in pairs, and when they decay they eventually decay down to an odd number of LSPs. If this decay is prompt (and the LSP is neutral) they leave a missing energy signature in detectors.

This idea of parity oddness for new particles is so successful that it has been borrowed many times for other BSM scenarios *e.g.* KK-parity leading to LKPs of extra dimensions,[7] T-parity and LTPs of Little Higgs scenarios.[8] In the rest of these lectures we will assume that R-parity is an exact symmetry of the MSSM but it is also possible that it is broken, that there is another symmetry that protects protons from decay or that the κ are tuned to be small.[9,10] If this were the case then SUSY would lose its dark matter candidate and depending on the timescale for decay its missing energy signature in colliders. To see how small the couplings to the light quarks would have to be consider the case of κ_1 and κ_4 non-zero, then there would be a tree-level diagram, involving squark exchange, that would lead to proton decay. Although an exact calculation is complicated, we need to know the details of the quark make-up of the proton, we can estimate the proton lifetime,

$$\tau^{-1} = \Gamma \sim \frac{|\kappa_1\kappa_4|^2}{16\pi}\frac{m_p^5}{m_{\tilde{q}}^4} \Rightarrow \tau \approx |\kappa_1\kappa_4|^{-2}\left(\frac{m_{\tilde{q}}}{1\ \text{TeV}}\right)^4 \times 10^{-11}s\ . \tag{78}$$

The proton lifetime is at least $\sim 10^{32}$ years implying that the relevant κ have to be very small, $|\kappa| \lesssim 10^{-12}$.

Now that we have forbidden the bad renormalisable operators[g] we have a fully supersymmetric version of the SM. A parameter count shows that the number of parameters is one smaller than that in the SM since the Higgs potential is entirely determined by the D-terms; we will discuss this in more detail in Sec. 5. However, this is not a fully realistic model since we know that the superpartners are not degenerate with their SM cousins. To break this degeneracy requires us to break SUSY and will introduce a

[g]There are higher dimension operators, such as $QQQL$, that can contribute in loops to proton decay. Depending on the scale that suppresses these operators they too can be a concern in supersymmetric theories.[11]

multitude (105!) of new parameters.[12] SUSY is great, breaking it is where the trouble begins.

4. The MSSM - broken SUSY

The supertrace condition Eq. (65) on tree-level SUSY breaking predicts superpartners lighter than the heaviest SM particle in each charge sector of the SM, *i.e.* sleptons are lighter than the tau, squarks are lighter than the top etc. This is clearly ruled out, which leads to the typical scenario for introducing SUSY breaking into the MSSM. We introduce some hidden sector whose dynamics is such that the vacuum of this sector is not supersymmetric[13] but is sufficiently heavy that the supertrace condition is not a concern. There are then some "messenger" fields which couple the MSSM to the dynamical SUSY breaking sector. The SUSY breaking in this dynamical SUSY breaking sector is then mediated to the MSSM through the messengers. For instance in gauge mediation[14] the messengers have SM gauge quantum numbers, whereas in gravity mediation the messenger fields are unspecified fields whose mass is at the Planck scale. Integrating out the messenger fields results in couplings between the SUSY breaking and the MSSM, the size of these couplings depends on the details of the mediation mechanism - a subject worthy of a series of TASI lectures itself.[15] However, the list of SUSY breaking operators is finite and one can parametrise all possibly combinations by considering just these operators, which we do below.

4.1. *Spurions*

For the purpose of these lectures, and much of SUSY phenomenology, it is sufficient to carry out a "spurion" analysis. Spurion analyses are a useful tool when one wishes to keep track of how a symmetry is broken, any parameter that breaks a symmetry can be elevated to the status of a field and the symmetry restored by assigning the appropriate transformation properties to the field. The field is not dynamical, its sole purpose is to get a vacuum expectation value which breaks the symmetry, restoring the parameter to the Lagrangian, but in so doing it helps us keep track of allowable operators. We have seen this before in the SM, at energies below the W mass we write down the QCD Lagrangian including mass terms for the quarks. But in reality, once we learn about $SU(2)_W$, we realise that these quark masses break $SU(2)_W$ which can be restored if the mass is thought of as transforming under the $SU(2)_W$ symmetry. In this case the spurion is nothing more than the SM Higgs that we soon hope to discover. In SUSY there need not be a physical particle associated with the spurion or it may be too heavy to ever be accessible but the restoration of SUSY will still be a useful tool.

In SUSY the available spurions whose VEV break the symmetry without also breaking Lorentz invariance are the F-term of a chiral superfield, $X = \theta^2 F$, or the D-term of a, $U(1)$, vector superfield, $W'_\alpha = \theta_\alpha D$. With these in hand we can ask what are the leading operators involving these spurions that will lead to SUSY breaking terms in the Lagrangian. In the MSSM the only relevant spurion is X and the important operators, generated at the messenger scale (M), are:

Scalar mass

$$c^{ij} \int d^4\theta \frac{X^\dagger X}{M^2} Q_i^\dagger Q_j \ , \tag{79}$$

which leads to a scalar mass^2 term in the Lagrangian of

$$-\left(m^2\right)^{ij} \tilde{q}_i^* \tilde{q}_j \ , \tag{80}$$

with $\left(m^2\right)^{ij} = -c^{ij}(F_X/M)^2$. This operator exists whether X is a MSSM singlet or not. The c^{ij} can have new flavour structure and if the sfermions are not well above the weak scale this can potentially lead to visible flavour violating effects, we will discuss this more in Sec. 6.1. Certain mediation mechanisms, for instance gauge mediation, predict that $c^{ij} \propto \delta^{ij}$ which avoids this problem. In gravity mediation there is no such prediction but nonetheless it is often assumed that the scalar masses generated at the Planck scale are flavour diagonal, primarily to avoid these strong constraints.

Gaugino mass

$$\frac{1}{2} c_i \int d^2\theta \frac{X}{M} W^\alpha W_\alpha \ , \tag{81}$$

which leads to a Majorana gaugino mass term in the Lagrangian of

$$-\frac{1}{2} m_i \tilde{\lambda}^\alpha \tilde{\lambda}_\alpha \tag{82}$$

where i here runs over the three gauge groups of the MSSM, and $m_i = -c_i F/M$. This operator can only be written down if X is a MSSM singlet. If this is not the case one would expect the scalar masses Eq. (79) to be far larger than the gaugino masses Eq. (81).

A term

$$\int d^2\theta \frac{X}{M} \left(A_u^{ij} U_i^c Q_j H_u - A_d^{ij} D_i^c Q_j H_d - A_e^{ij} E_i^c L_j H_d \right) \ , \qquad (83)$$

which leads to scalar trilinear terms in the Lagrangian of

$$a_u^{ij} \tilde{u}_i^c \tilde{q}_j h_u - a_d^{ij} \tilde{d}_i^c \tilde{q}_j h_d - a_e^{ij} \tilde{e}_i^c \tilde{\ell}_j h_d \qquad (84)$$

with $a^{ij} = A^{ij} F/M$. As for the gaugino masses this operator requires that the spurion is a MSSM singlet. Furthermore, the A terms are another new source of flavour violation and so have strong constraints on the sizes of the flavour off-diagonal terms.

b **term**

$$B \int d^4\theta \frac{X^\dagger X}{M^2} H_u H_d \ , \qquad (85)$$

which leads to a scalar mass^2 term in the Lagrangian of

$$-b h_u h_d \ , \qquad (86)$$

with $b = -B(F/M)^2$. If X is a singlet then a μ term, a supersymmetric parameter, can also be generated from $X^\dagger H_u H_d / M$ in the Kähler potential. For successful electroweak symmetry breaking, as we will discuss further in Sec. 5, the supersymmetric mass parameter, μ, must be around the weak scale and the SUSY preserving and breaking parameters related by $b \sim \mu^2$. If both these two operators are generated with comparable coefficients, as can occur, for example, in gravity mediated theories,[16] then this provides a solution to the μ-b problem.

These are the leading operators discussed in the context of the MSSM. There are higher dimension operators that are typically generated with small coefficients at the messenger scale. In the absence of MSSM gauge singlets these additional operators have also been shown[17] to be "soft" [h]. They correspond to non-holomorphic combinations of MSSM fields, *e.g.* $X^\dagger Q H_u^\dagger D^c / M^2$ and $X^\dagger X Q H_u^\dagger D^c / M^3$. Although they are typically small at the messenger scale these operators may be generated through renormalisation group running and can lead to interesting "wrong-type" Higgs couplings.

If the field content of the MSSM is extended then there are more operators that can be written down. One interesting possibility, that uses a D-term spurion, is that of supersoft SUSY breaking.[18] The MSSM is

[h] They don't generate quadratic divergences, only logarithmic ones.

extended by adding chiral superfields, A^i, that transform in the adjoint representation of $U(1)$, $SU(2)$ and $SU(3)$, for $i = 1, 2, 3$, respectively. This now allows us to write down Dirac gaugino mass terms:

Supersoft term

$$\sqrt{2} \int d^2\theta \frac{W'^{\alpha}}{M} W_{\alpha}^i A^i \ , \tag{87}$$

which results in a gaugino-adjoint Dirac mass, a mass term for (the real part of) the scalar adjoint, and a scalar tri-linear term,

$$-m_D \tilde{\lambda}_i \tilde{a}_i - m_D^2 (a_i + a_i^*)^2 - \sqrt{2} m_D (a_i + a_i^*) \left(\sum_j g_k q_j^* t_a q_j \right) \ , \tag{88}$$

where $m_D = D'/M$ and q represents all MSSM fields charged under gauge group i. Models with just a D-term spurion have interesting renormalization properties.[18–24]

4.2. *Supersymmetry breaking scenarios*

Many of the operators described above have coefficients that are complex 3×3 matrices in flavour space, so the full list of soft-SUSY breaking operators possible in the MSSM is long,[12] all-in-all $\mathcal{O}(100)$ scales, angles and phases are involved. This is obviously too vast a parameter space to explore fully, and given constraints from experiment we know that much of it, *e.g.* parts with large flavour violation, is already ruled out. Luckily most of the mediation mechanisms relate many of the operators, and the soft SUSY breaking terms are defined by a handful of parameters at the messenger scale.

One common simplifying assumption, when assuming gravity mediation, is that there are common scalar and fermion masses, and that the A terms are proportional to the corresponding Yukawas with the same constant of proportionality. At the messenger scale, M_{Pl}, this means,

$$\begin{aligned}
&m_1 = m_2 = m_3 = m_{1/2} \\
&m_{\tilde{q}}^2 = m_{\tilde{u}^c}^2 = m_{\tilde{d}^c}^2 = m_{\tilde{\ell}}^2 = m_{\tilde{e}^c}^2 = m_0^2 \mathbb{1} \text{ and } m_{h_u}^2 = m_{h_d}^2 = m_0^2 \\
&a_u = a_0 Y_u, \ a_d = a_0 Y_d, \ a_L = a_0 Y_L \\
&b = b_0 \, \mu \ .
\end{aligned} \tag{89}$$

This common assumption often goes under the name of minimal supergravity (MSUGRA) or constrained MSSM (CMSSM). As we will see later, the requirement of correct electroweak symmetry breaking, after using the renormalisation group evolution of MSSM parameters, leads to this boundary condition often being defined in terms of m_0, $m_{1/2}$, a, $\tan\beta$ and $\mathrm{sgn}\,\mu$.

This boundary condition shrinks parameter space from ~ 100 dimensional to 5 dimensional since it is also usually assumed that all the scales are real, the 5 parameters are: m_0^2, $m_{1/2}$, a_0, b_0, μ. This combined with the flavour structure of the scalar masses^2 and A terms avoids flavour and CP-violation constraints. Since the messenger scale is M_{Pl} the F-term of the SUSY breaking spurion must be at the intermediate scale $F^{1/2} \sim 10^{10} - 10^{11}$ GeV so that $m_{1/2} \sim F/M_{Pl} \sim 100$ GeV.

In gravity mediation there is little top-down motivation that explains the arrangement of diagonal scalar mass^2, or A terms. Gauge mediation, on the other hand, is flavour blind and so by itself does not lead to a problem with FCNCs. If the messengers are at the Planck scale then one would expect that in addition to flavour-diagonal gauge mediated contributions there will also be flavour-violating gravity mediated contributions. Thus, gauge mediation is usually assumed to have light messengers ($M \gtrsim 100$ TeV) in order to avoid flavour issues. In gauge mediation the gaugino masses are generated at one loop and the scalar masses^2 at two loop so that the actual mass scales are comparable. Furthermore, the A terms, which have mass dimension one, are generated at two loops and so are negligible. Thus the boundary condition at the messenger scale, assuming the messengers are simply a $\mathbf{5} + \overline{\mathbf{5}}$ pair - so called minimal gauge mediation, for the gauginos (m_i) and a scalar (m_j^2) are:

$$\begin{aligned} m_i &= \frac{\alpha_i}{4\pi}\frac{F}{M} \\ m_j^2 &= 2\left(\frac{F}{M}\right)^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3(j) + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2(j) + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1(j)\right] \\ a_u &= a_d = a_e = 0 \ . \end{aligned} \tag{90}$$

The $C_i(j)$ are the quadratic Casimirs for each group, they are given by $(N^2-1)/2N$ for $SU(N)$ and $3Y^2/5$ for hypercharge and 0 if the scalar is not charged under that group. Notice that I have not specified the values for μ or b since these terms are not generated by minimal gauge mediation. For low scale gauge mediation the SUSY breaking F-term is far smaller than that in gravity mediation, $F^{1/2} \sim 10^4$ GeV. See Patrick Meade's lectures in this volume for a more detailed discussion of gauge mediation Ref. 14.

So far we have been agnostic as to the source of the supersymmetry breaking F-term contained in the spurion, not an unreasonable approach since we are most interested in learning about MSSM phenomenology. One possibility for the origin of SUSY breaking is an O'Raifeartaigh model of the type described in Sec. 2.5. However, in these models the scale of SUSY breaking is determined by the scale appearing in the superpotential, which is put in by hand and, although explaining the stability of the weak scale, it does not explain its origin *i.e.* why the weak scale is so far below the Planck, or GUT, scale. It would be nice if there were some mechanism whereby a scale far below the GUT scale is generated dynamically and which is involved in SUSY breaking.

We are familiar with a low scale, Λ, being generated from a theory that is perturbative at some high scale, through the phenomenon of dimensional transmutation. For example, in QCD non-perturbative dynamics generates a scale from a theory that is weakly coupled at the Planck scale, $\Lambda \sim M_{Pl} e^{-8\pi^2/g^2}$, and this scale is far below the Planck scale due to the exponential. Similar non-perturbative corrections to the superpotential of a theory that is classically supersymmetric can lead to SUSY being broken dynamically, and the associated scale is far below the Planck scale, potentially explaining the origin of the electroweak scale. Many examples of dynamical supersymmetry are known, it is a rich subject that will be discussed in greater detail in David Shih's lectures.[13]

4.3. *The Goldstino and gravitino*

In Sec. 2.5 we analysed a simple O'Raifeartaigh model of SUSY breaking and found that there is a massless Weyl fermion in the spectrum, we now extend this result to a general SUSY gauge theory in which SUSY is spontaneously broken. The fermion mass matrix in the basis (λ^a, ψ_i) is,

$$M_{fermion} = \begin{pmatrix} 0 & \sqrt{2}\langle\phi^*\rangle T^a \\ \sqrt{2}\langle\phi^*\rangle T^a & \langle W^{ij}\rangle \end{pmatrix} \tag{91}$$

where T^a are the generators of the gauge group and $W^{ij} = \partial^2 W/\partial\Phi_i \partial\Phi_j$. This mass matrix has an eigenvector with zero eigenvalue, namely

$$\tilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix} \ . \tag{92}$$

Exercise: Show that Eq. (92) is indeed a zero eigenvector. It is useful to recall the condition for gauge invariance of the superpotential, and that in the ground state $\partial V/\partial\phi_i = 0$.

This massless fermion is the Goldstino, it is built from fields of the SUSY breaking sector, and will always be in the spectrum when SUSY is spontaneously broken. The Goldstino is entirely analagous to the Goldstone boson of a spontaneously broken bosonic symmetry, but is a fermion because the symmetry being broken has fermionic generators. Just as the Goldstone boson will be eaten, by the gauge boson making it massive, if the global symmetry is gauged so the Golsdstino will be eaten if SUSY is extended to a local symmetry. Since SUSY is a spacetime symmetry making it a local symmetry necessitates gauging of Poincare symmetry and so introduces gravity and turns supersymmetry into supergravity. The supersymmetric partner of the spin 2 graviton is a spin 3/2 gravitino. This gravitino, $\tilde{\Psi}_\mu^\alpha$, will eat the Goldstino and become massive. The mass of the gravitino is related to the breaking of SUSY[25] ,

$$m_{3/2}^2 = \frac{|F|^2 + \frac{1}{2}D^2}{3M_{Pl}^2} \ . \tag{93}$$

For gravity mediated SUSY breaking the messenger scale is M_{Pl} and comparing Eq. (90) and Eq. (93) we see that the gravitino is comparable in mass to the SM superpartners. For gauge mediation the messenger scale is far below M_{Pl} and the gravitino is far lighter, $m_{3/2} \gtrsim 1$ eV, than the SM superpartners. From Eq. (77) we see that the gravitino is odd under R-parity and so in gauge medition is the LSP and will be the final endpoint of all superpartner decays.

5. Higgs sector of the MSSM

Due to holomorphy of the superpotential the MSSM is a type II two Higgs doublet model *i.e.* one Higgs, H_u, couples to up-type quarks and the other, H_d, couples to down-type quarks and leptons Eq. (75). Furthermore, SUSY enforces various relations between the parameters of the general type II scalar potential, for a discussion about general Higgs models see, for example, Ref. 26.

5.1. *Electroweak Symmetry Breaking*

There are three contributions to the Higgs scalar potential: those from F terms, D terms and SUSY breaking,

$$V_{Higgs} = V_F + V_D + V_{soft} \ . \tag{94}$$

From Eq. (34) and Eq. (75) we find that,

$$V_F = |\mu|^2 \left(|h_u|^2 + |h_d|^2\right) \ . \tag{95}$$

There are D-term contributions Eq. (45) from both $U(1)_Y$ and $SU(2)_W$,

$$D_2^a = -g\,(h_u^* \tau^a h_u + h_d^* \tau^a h_d) \quad D_1 = -\frac{g'}{2}\left(|h_u^+|^2 + |h_u^0|^2 - |h_d^0|^2 - |h_d^-|^2\right)\ , \tag{96}$$

where $\tau^a = \sigma^a/2$ are the generators of $SU(2)_W$, $g' = e/\cos\theta_W$ and $g = e/\sin\theta_W$. Using Eq. (46), and carrying out some algebra, this gives

$$V_D = \frac{g^2 + g'^2}{8}\left(|h_u^+|^2 + |h_u^0|^2 - |h_d^0|^2 - |h_d^-|^2\right)^2 + \frac{g^2}{2}\left|h_u^+ h_d^{0*} + h_u^0 h_d^{-*}\right|^2\ . \tag{97}$$

Exercise: Confirm that Eq. (96) does indeed lead to Eq. (97).

Finally there are the soft terms generated by SUSY breaking,

$$V_{soft} = m_{h_u}^2 |h_u|^2 + m_{h_d}^2 |h_d|^2 + (b\,\epsilon_{\alpha\beta}\, h_u^\alpha h_d^\beta + h.c.)\ . \tag{98}$$

If electroweak symmetry breaking is to work correctly in the MSSM then the vacuum must break $SU(2)_W \times U(1)_Y$ to $U(1)_Q$, which means the charged Higgses must not get a VEV. To see that this can indeed happen note that one can carry out an $SU(2)_W$ rotation on H_u such that any VEV in the scalar lies entirely in the neutral component. Then the minimisation condition $\partial V/\partial h_u^+ = 0$ evaluated at the minimum with $\langle h_u^+\rangle = 0$ implies that $\langle h_d^-\rangle = 0$, a good thing. This means that we can concentrate on just the neutral components of the Higgses,

$$\begin{aligned} V = &\left(|\mu|^2 + m_{h_u}^2\right)|h_u|^2 + \left(|\mu|^2 + m_{h_d}^2\right)|h_d|^2 \\ &-(b\, h_u^0 h_d^0 + h.c.) + \frac{g'^2 + g^2}{8}\left(|h_u^0|^2 - |h_d^0|^2\right)^2\ . \end{aligned} \tag{99}$$

Also, it is always possible to make a field rotation such that b is real and positive, which means that $\langle h_u^0 h_d^0\rangle$ is also real and positive. Thus, the two Higgses must have VEVs with equal and opposite phase, which can be set to zero by carrying out a $U(1)_Y$ rotation since they transform with opposite sign under hypercharge. The upshot of all of this is that in the MSSM CP and electric charge are not spontaneously broken.

To be sure that electroweak symmetry is indeed broken at least one of the Higgs bosons must acquire a VEV which means the origin cannot be a stable minimum of the potential. We look at the mass matrix near the origin,

$$\left(h_u^{0*}\ h_d^{0*}\right)\mathcal{M}_h^2\begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \left(h_u^{0*}\ h_d^{0*}\right)\begin{pmatrix} |\mu|^2 + m_{h_u}^2 & -b \\ -b & |\mu|^2 + m_{h_d}^2 \end{pmatrix}\begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix}\ , \tag{100}$$

in order for the origin to be unstable the mass matrix, $\mathcal{M}_h^2$, must have either one or two negative eigenvalues. These two possibilities correspond to $\det \mathcal{M}_h^2 < 0$ or $\det \mathcal{M}_h^2 > 0$ and $\mathrm{tr}\, \mathcal{M}_h^2 < 0$ respectively. In addition to the origin being unstable we also require that the potential does in fact have a minimum, *i.e.* is is not unbounded from below. Clearly the quartic term is always positive and at generic large field value it will stabilise the potential. However, there is a special direction in field space, called a D-flat direction, in which the quartic term disappears, $h_u^0 = h_d^0$. The requirement that along this direction the potential does not run away is,

$$2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2 > 2b \ . \tag{101}$$

Since $b > 0$ this rules out the possibility of $\mathrm{tr}\mathcal{M}_h < 0$. Instead we will have one negative eigenvalue if,

$$(|\mu|^2 + m_{h_u}^2)(|\mu|^2 + m_{h_d}^2) < b^2 \ . \tag{102}$$

Taken together Eq. (101) and Eq. (102) are the requirements for correct electroweak symmetry breaking. For a non-zero b term both Higgses will acquire a vev,

$$\langle h_u^0 \rangle = v_u \qquad\qquad \langle h_d^0 \rangle = v_d \ , \tag{103}$$

and electroweak symmetry will be broken. To get the correct W and Z masses requires that the VEVs satisfy the relation,

$$v_u^2 + v_d^2 = v^2 = \frac{2M_Z^2}{g'^2 + g^2} \approx (174 \text{ GeV})^2 \ . \tag{104}$$

One can define the ratio of the VEVs as $\tan\beta \equiv v_u/v_d$. The minimisation conditions then lead to two conditions on the various parameters of the tree-level potential,

$$\begin{aligned} m_{h_u}^2 + |\mu|^2 - b\cot\beta - \frac{M_Z^2}{2}\cos 2\beta = 0 \\ m_{h_d}^2 + |\mu|^2 - b\tan\beta + \frac{M_Z^2}{2}\cos 2\beta = 0 \ . \end{aligned} \tag{105}$$

This again illustrates the μ-problem, since satisfying Eq. (105) requires the SUSY breaking parameters $m_{h_u}^2$, $m_{h_d}^2$ and b are related to the SUSY preserving parameter μ. Without precise cancellations they should all be around the weak scale, which is reasonable for SUSY breaking parameters but the SUSY preserving μ-term could have value anywhere between the weak and Planck scales. Using these conditions also relates various SUSY breaking parameters so that the SUSY breaking inputs of Eq. (89) are often quoted as m_0, $m_{1/2}$, A, $\tan\beta$ and $\mathrm{sgn}(\mu)$, which is four parameters and a discrete choice.

The region of viable Higgs soft-mass parameter space is shown in Fig. 4, the region is contained between the two coloured lines. Above the upper (blue curved) line the Higgs mass matrix has only positive eigenvalues and the Higgses do not get a VEV, corresponding to violating Eq. (102). While below the lower (red straight) line the Higgs potential is unbounded from below and the Higgs potential has no stable minimum, violating Eq. (101). Thus, the interesting space lies between these two curves and is separated into two distinct regions. The region with large $m_{h_d}^2 + |\mu|^2$ and small $m_{h_u}^2 + |\mu|^2$ has $\tan\beta \geq 1$, Eq. (105), and the other region has $\tan\beta \leq 1$; lines of constant $\tan\beta$ are dashed in the plot. As we will see shortly correct fermion masses require us to be in the $\tan\beta \geq 1$ region.

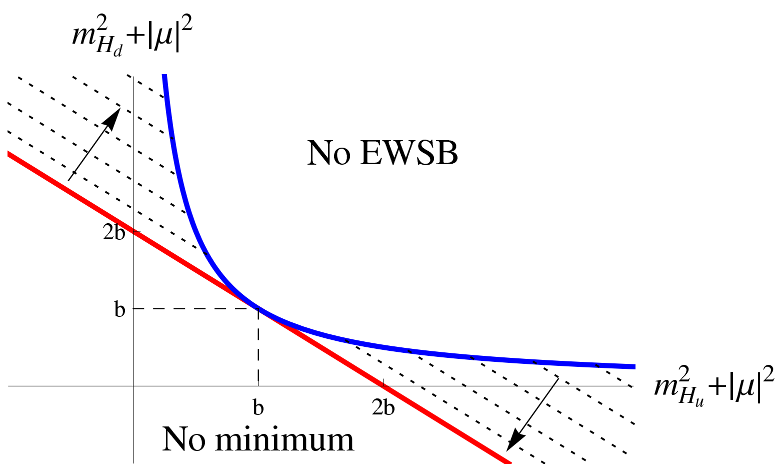

Fig. 4. The viable region of Higgs soft-mass parameter space lies between the red (straight solid) and blue (curved solid) lines, corresponding to the bounds Eq. (101) and Eq. (102) respectively. Above the blue line there is no electroweak symmetry breaking and below the red line the potential is unbounded from below. The dashed lines are lines of constant $\tan\beta$ and the arrows denote the direction in which $\tan\beta$ increases, the red line corresponds to $\tan\beta = 1$.

5.2. *Higgs masses*

We are now in a position to talk about the Higgs bosons in the MSSM. With two doublets there are 8 real degrees of freedom, 3 of which are eaten (just as in the SM) to give mass to the $W^\pm$ and Z, leaving 5 physical degrees of freedom. These are two real scalars, h^0 and H^0, with h^0 lighter than H^0, a pair of charged Higgses $h^{\pm}$ and one CP odd scalar, A^0. Their masses are

$$\begin{aligned} M_{A^0}^2 &= \frac{2b}{\sin 2\beta} = 2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2 \\ M_{h^\pm}^2 &= M_{A^0}^2 + M_W^2 \\ M_{h^0,H^0}^2 &= \frac{1}{2}\left(M_{A^0}^2 + M_Z^2 \mp \sqrt{\left(M_{A^0}^2 - M_Z^2\right)^2 + 4M_Z^2 M_{A^0}^2 \sin^2 2\beta}\right) \end{aligned} \tag{106}$$

Notice that the masses of the heavy Higgses, *i.e.* all Higgses all except h^0, scale with the mass of the psuedoscalar. This leads to a particular limit, $M_{A^0} \gg M_Z$ dubbed the decoupling limit, in which the heavy Higgses form a nearly degenerate $SU(2)$ doublet and decouple from low energy physics, the remaining light Higgs, h^0, behaves very much like the Higgs of the SM. A large region of SUSY parameter space is in the decoupling limit.

Exercise: Expand Eq. (99) around the minimum,

$$h_u = \begin{pmatrix} h_u^+ \\ v_u + h_u^0 \end{pmatrix} \quad h_d = \begin{pmatrix} v_d + h_d^0 \\ h_d^- \end{pmatrix} \ , \tag{107}$$

and show that the masses for the Higgses are given by Eq. (106). Work out the mass eigenstates in terms of the gauge eigenstates.

So far we have been working at tree level, but as we can see from Eq. (106) this is already ruled out by LEP Higgs searches since the mass of the lightest Higgs, h^0, is bounded above by,

$$M_{h^0} \leq M_Z |\cos\beta| \ . \tag{108}$$

This is because in supersymmetric theories the Higgs self-couplings are determined by the D-terms and are related to the gauge couplings of the SM, but once SUSY is broken there will be corrections. The largest corrections will come from particles that have the largest coupling to the Higgs, typically the top and stop. There are one-loop diagrams involving the stop and top that will give corrections to the Higgs quartic term in the potential, these diagrams would of course cancel if SUSY was unbroken and the stop was degenerate with the top, it is the partial failure of this cancellation that leads to the corrections. We will see in Sec. 6 that there are two stop states that may have large mixing, which complicates the calculation of this correction, but approximately speaking,

$$M_{h^0}^2 \to M_{h^0}^2\big|_{tree} + \frac{3}{4\pi^2} y_t^2 m_t^2 \log\left(\frac{M_{\tilde{t}}^2}{M_t^2}\right) , \tag{109}$$

this raises the upper bound on the Higgs which now depends on the mass of the stop squark. However, in addition to this loop correction there are corrections to the Higgs soft mass term that are logarithmically sensitive to the SUSY breaking scale, M, at which superpartner masses are generated,

$$\delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2} M_{\tilde{t}}^2 \log\left(\frac{M}{M_{\tilde{t}}}\right) . \tag{110}$$

Thus, one cannot make the stops too heavy in order to raise the Higgs mass without reintroducing the hierarchy problem that SUSY was first introduced to solve. This competition between logarithmic and quadratic sensitivity to the stop mass, and the requirement of evading the LEP bound on the Higgs mass leads to the "SUSY little hierachy problem".[27] There are many proposed solutions to this problem, all of which require extending the MSSM, see for example.[28–41] If instead we limit the amount of tuning and impose an upper bound on the stop mass of ~ 1 TeV then there is a new upper bound on the Higgs mass of,

$$M_{h^0} \lesssim 130 \text{ GeV} . \tag{111}$$

5.3. *Higgs Yukawa couplings*

The values of the Yukawa couplings in the MSSM are different from those in the SM[i]. Concentrating on the two heaviest quarks, the masses are related, at tree level, to the superpotential Yukawa terms by,

$$m_t = y_t v_u = y_t v \sin\beta = y_t v \frac{\tan\beta}{\sqrt{1+\tan^2\beta}} \tag{112}$$

$$m_b = y_b v_d = y_b v \cos\beta = y_b v \frac{1}{\sqrt{1+\tan^2\beta}} . \tag{113}$$

Requiring the Yukawa couplings for top and bottom both remain perturbative places limits on the range of $\tan\beta$. As we will see, the renormalization group evolution of these couplings is such that their values tend to increase from their value at the weak scale, determined by Eq. (113), as we run

[i]Since one Higgs is responsible for up-type quark masses and one for down-type quarks and leptons it appears that both must acquire a VEV. For an alternative approach, where at tree level only $\langle h_u \rangle \neq 0$, see Ref. 42.

up to the GUT scale. Requiring that these couplings remain perturbative all the way up to the GUT scale, a strong but not entirely unreasonable requirement, results in the range $1 \lesssim \tan\beta \lesssim 60$.

6. Superpartner Mass Spectra

We now turn to the masses of the superpartners. After electroweak symmetry breaking the neutral components of the Higgsinos, $\tilde{h}_u$ and $\tilde{h}_d$, will mix with the bino, $\tilde{B}$, the partner of the $U(1)_Y$ gauge boson and the wino, $\tilde{W}^0$, the partner of the neutral component of the $SU(2)_W$ gauge boson. The mass eigenstates are called neutralinos and are variously denoted as $\tilde{N}_i$, $\tilde{\chi}_i^0$, $\tilde{Z}_i$, but in all cases i labels the mass eigenstates from lightest, $i = 1$ to heaviest $i = 4$. Extensions of the MSSM will naturally have more neutralinos.

We define $\Psi_0^T = (\tilde{B}, \tilde{W}, \tilde{h}_d, \tilde{h}_u)$ which has a mass term in the Lagrangian of $-\frac{1}{2}\Psi_0^T M_N \Psi_0 + c.c.$ with

$$M_N = \begin{pmatrix} m_1 & 0 & -c_\beta s_W M_Z & s_\beta s_W M_Z \\ 0 & m_2 & c_\beta c_W M_Z & -s_\beta c_W M_Z \\ -c_\beta s_W M_Z & c_\beta c_W M_Z & 0 & -\mu \\ s_\beta s_W M_Z & -s_\beta c_W M_Z & -\mu & 0 \end{pmatrix} \tag{114}$$

The real eigenvalues of this complex symmetric matrix can be found by diagonalising, $M_N^{diag} = U^* M_N U^\dagger$. If the off diagonal terms due to electroweak symmetry breaking are small relative to the other entries in the matrix then the mixing is small and the lightest neutralino will be mostly bino-, Higgsino- or Wino-like.

The charged components of the same fields will also mix. Define $\Psi_\pm^T = (\tilde{W}^+, \tilde{h}_u^+, \tilde{W}^-, \tilde{h}_d^-)$ which has a Lagrangian term $-\frac{1}{2}\Psi_\pm^T M_C \Psi_\pm$ with

$$M_C = \begin{pmatrix} 0 & 0 & m_2 & \sqrt{2}s_\beta M_W \\ 0 & 0 & \sqrt{2}c_\beta M_W & \mu \\ m_2 & \sqrt{2}c_\beta M_W & 0 & 0 \\ \sqrt{2}s_\beta M_W & \mu & 0 & 0 \end{pmatrix} . \tag{115}$$

It is easier to work with the non-trivial 2×2 block of M_C, M. Since M is not symmetric it is diagonalised by two different unitary transformations, $M^{diag} = L^* M R^\dagger$. The masses of the charginos, denoted by $\tilde{C}_i$, $\tilde{\chi}_i^\pm$ or $\tilde{W}_i^\pm$, are given by

$$\begin{aligned} M_{C_1,C_2} = \ & \frac{1}{2}\Big[|m_2|^2 + |\mu|^2 + 2M_W^2 \\ & \mp\sqrt{\left(|m_2|^2 + |\mu|^2 + 2M_W^2\right)^2 - 4|\mu m_2 - M_W s_{2\beta}|^2}\Big] \ . \end{aligned} \tag{116}$$

Given the issues with generating the correct size for the μ-term, discussed earlier, one might wonder if these problems are removed if the μ-term is somehow forbidden. However, notice that $\mu = 0$ would lead to both a massless neutralino and a chargino below the W mass, which is ruled out by LEP searches. However, extensions of the MSSM can be built that do not have a μ-term,[43] or where the μ-term is generated from SUSY breaking. The later case is a commonly discussed extension of the standard model: the exension of the MSSM by a gauge singlet chiral superfield, S, called the next-to-minimal supersymmetric standard model (NMSSM).

Exercise: By finding the eigenvalues of $M^\dagger M$ confirm Eq. (116).

The sfermion masses receive contributions from various sources. I will discuss the case of the stops, the other squarks and sleptons masses follow in analogous fashion, and I will also ignore, for now, potential flavour violating contributions to the mass matrices. Working in the basis $\Psi_t^T = (\tilde{q}_3, \tilde{u}_3^c)$ the stop mass matrix is

$$M_t = \begin{pmatrix} M_t^2 + m_{\tilde{q}_3}^2 + \Delta_{\tilde{q}_3} & M_t(A_t^* - \mu\cot\beta) \\ M_t(A_t - \mu\cot\beta) & M_t^2 + m_{\tilde{u}_3^c} + \Delta_{\tilde{u}_3^c} \end{pmatrix} \ . \tag{117}$$

The soft scalar masses $m_{\tilde{q}_3}^2$ and $m_{\tilde{u}_3^c}$ arise from SUSY breaking as described in Eq. (79). The $\Delta_{\tilde{f}}$ terms arise from the $SU(2)_W$ and $U(1)_Y$ D-terms in the scalar potential. For example the relevant piece of the $U(1)_Y$ D-term is $-\frac{g'}{2}\left(|h_u^0|^2 - |h_d^0|^2 + \sum_i \tilde{f}_i^* Y_i \tilde{f}_i\right)$. In general for a sfermion, $\tilde{f}$, the D-term contributions to the mass matrix are

$$\Delta_{\tilde{f}} = \left(T_3 - Q\sin^2\theta_W\right)_{\tilde{f}} \cos 2\beta M_Z^2 \ . \tag{118}$$

For third generation sfermions like the top there are F-term contributions, from the F-terms for U_3, Q_3 and H_U, these give the contributions proportional to μ in Eq. (117). Finally there are A-term contributions (see Eq. (84)) where in Eq. (117) I have followed the oft-used convention of $a_i = y_i A_i$. Similar matrices exist for the other squark and sleptons, however for down-type squarks and sleptons the down-type Higgs F-terms is involved, thus one must also make the replacement $\tan\beta \leftrightarrow \cot\beta$. The mass matrix Eq. (117) must be diagonlised and the resulting mass eigenstates are denoted $\tilde{t}_1$ and $\tilde{t}_2$ with the convention that $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$.

The only remaining superpartner left to discuss is the gluino. Since it is the only octet of colour it has nothing to mix with and its mass is simply given by M_3.

6.1. *Flavour and CP violation*

In general the squark and slepton masses are 3×3 matrices in flavour space, and there is no reason *a priori* for the squark and slepton masses to be diagonal in the same basis as the quark and lepton Yukawa matrices. These mass matrices introduce a new source of flavour violation beyond the flavour violation in the SM which comes from the CKM matrix, and neutrino mixing. Furthermore, there is no reason for all the entries in these matrices to be real, or for the phases in other SUSY parameters such as gauginos masses to be zero. Thus there are additional sources of CP violation introduced by the MSSM.

These sources of flavour and CP violation can lead to observable effects, such as flavour changing neutral currents (FCNCs) and electric dipole moments (EDMs), respectively. Searches for FCNCs and EDMs in various systems place strong constraints on the size of the off-diagonal entries in the squark and slepton mass matrices and the size of the CP-violating phases in MSSM parameters. Let us consider two such processes.

Although FCNCs do not occur at tree-level in the SM they can occur at one loop. In Fig. 5, we consider the case of $K^0 - \bar{K}^0$ mixing, occurring through box diagrams. Since the CKM matrix is unitary the diagram vanishes when the quarks are massless. The leading contribution to $K - \bar{K}$ mixing in the SM is quadratic in quark mases, this is the GIM mechanism.[44] In Fig. 5 the fact that the leading effect is proportional to quark mass (squared) is denoted by the crosses on the internal quark lines. Since the quark masses are small compared to the W mass, the top has very small coupling to the first generation and is ignored here, it is useful to treat the quarks as having massless propagators and the mass as being a coupling that can be inserted on a fermion line, the mass insertion technique.[45] It is easy to see that the mass insertion picture is just a rewriting of the usual fermion propagator, but it is nonetheless a useful tool,

$$\frac{i}{\not{p} - m} = \frac{i}{\not{p}} + \frac{i}{\not{p}}(-im)\frac{i}{\not{p}} + \ldots \tag{119}$$

In the SM the effective four-quark operator generated by the box diagrams has coefficent

$$\alpha_W^2 \frac{m_c^2}{M_W^4} |V_{cs}|^2 |V_{cd}|^2 \ , \tag{120}$$

which has mass dimension -2 as expected.

Now turning to the MSSM, there will be additional contributions to the four-quark operator from diagrams that are the supersymmetric version of the SM diagram *i.e.* box diagrams involving Winos and squarks and these will be large unless there is a super-GIM mechanism, namely that the first and second generation squarks must be nearly degenerate. Furthermore there are additional diagrams, that are often larger since they only involve strong couplings, Fig. 5. Now, however, the flavour violation is a result of off-diagonal squark mass entries, denoted by a cross. Making a simplifying assumption that all superpartners have the same mass, M_{susy}, and denoting the off-diagonal entries by Δm^2, we can estimate the contribution of Fig. 5 to the four-quark operator,

$$\alpha_3^2 \left(\frac{\Delta m^2}{M_{susy}^2} \right)^2 \frac{1}{M_{susy}^2} \ . \tag{121}$$

Depending on whether the external fermions are left- or right-handed the mass insertions will come from $m_{\tilde{q}}^2$ or $m_{\tilde{u}}^2$, $m_{\tilde{d}}^2$. If there are off-diagonal A-terms then there will be LR squark mixing.

The observed mass splitting between K_L and K_S is approximately explained by SM effects and so this places a bound on the size of the MSSM contribution

$$\frac{\Delta m^2}{M_{susy}^2} \lesssim 10^{-3} \frac{M_{susy}}{500 \text{ GeV}} \ . \tag{122}$$

So without pushing the SUSY scale very high, and so removing the motivation for SUSY, we see that this bound requires small off-diagonal terms in the $(\tilde{d}, \tilde{s})$ block of the squark mass^2 matrix. Similar, but weaker, constraints exist for the other off-diagonal terms of the squark mass matrix[2]. These bounds are often quoted[46,47] as bounds on the dimensionless ratio $\delta \equiv \Delta m^2 / M_{susy}^2$.

In addition to these $\Delta F = 2$ processes in the quark sector, there are bounds on off-diagonal entries coming from $\Delta F = 1$ processes such as $b \to s\gamma$, $b \to s\ell^+\ell^-$, etc and $\mu \to e\gamma$. For more detail about flavour constraints on the MSSM and other theories of BSM physics see Gilad Perez's lectures in this volume.[48]

Exercise: The fact that neutrinos oscillate leads to lepton flavour violation. Assume there is maximal mixing between two flavours of neutrinos (ν_α and ν_β), a good approximation for reality, and estimate the induced branching ratio of $\ell_\alpha \to \ell_\beta \gamma$. Confirm that this is not an effect we need worry about for the foreseeable future.

There are many possibilities for new CP-violating phases in the parame-

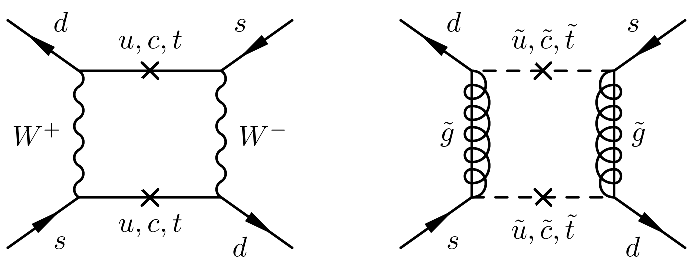

Fig. 5. SM and MSSM processes contributing to $K - \bar{K}$ mixing.

ters of the MSSM, the gaugino masses, the μ and b terms, and A-terms. Not all the phases are physical, some can be removed by various field rotations but there are still a plethora of new sources of CP violation. Inclusion of these physical phases alters the mass matrices and the corresponding mass eigenstates, *e.g.* Eq. (114) and Eq. (115), and will also affect the couplings of the superpartners.

These phases are further constrained by the experimental bounds on electric dipole moments (EDMs) of the electron and neutron. There are strong constraints on both EDMs, for instance the present bound[49] on the electron EDM coming from a search for T violation in ^{205}Tl is,

$$|d_e| \leq 1.6 \times 10^{-27} e \text{ cm} , \tag{123}$$

at 90% confidence. The phases in SUSY parameters can contribute to these EDMs. For instance, the electron EDM receives contributions from loop diagrams involving the insertion of a complex parameter, such as a complex A-term,Fig. 6. We can estimate the contribution of this to the EDM of the electron as,

$$d_e^{susy} \approx e \frac{\alpha}{4\pi} \frac{m_e}{\tilde{m}^2} \arg(A) . \tag{124}$$

Where I have assumed that the A-term is proportional to the Yukawa and that all SUSY scales are comparable and of size $\tilde{m}$. Requiring this to be smaller than the bound Eq. (123) leads to the requirement that

$$\arg(A) < 10^{-2} \left(\frac{\tilde{m}}{500 \text{ GeV}} \right)^2 . \tag{125}$$

CP violation is strongly constrained unless one wishes to raise the scale of the superpartners.

It seems that low energy constraints such as FCNCs and EDMs pick out a particular region of SUSY parameter space. Thankfully there are many models of SUSY breaking that predict we lie in exactly this region. See Patrick Meade's lectures in this volume for a detailed discussion of one of these mechanisms of SUSY breaking, gauge mediation.[14]

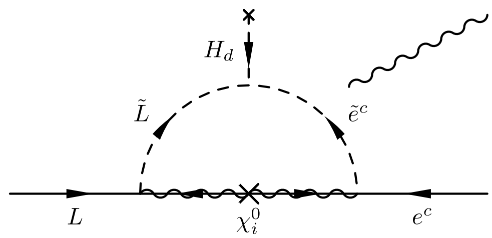

Fig. 6. MSSM contribution to electron EDM, the photon is attached to the loop in all possible ways.

7. RGEs

In Sec. 4 we discussed the generation of SUSY breaking parameters at a high scale, the messenger scale, whereas in Sec. 6 we discussed the masses of the superpartners at the weak scale. The natural tool for relating the two is renormalisation. I will only discuss one-loop renormalisation equations, for more details and two loop expressions see Ref. 50.

7.1. *Gauge couplings*

Since the normalisation of the generator of a $U(1)$ group is arbitrary, the physical quantity coupling $\times$ charge must remain fixed, we are free to rescale the $U(1)_Y$ gauge coupling. A convenient choice is $g_1 = \sqrt{5/3}g'$ which would be the normalisation if the $U(1)$ were embedded in $SU(5)$ or $SO(10)$, in addition we define $g_2 = g$ and $g_3 = g_s$. The one-loop renormalisation group equations which describe how the gauge couplings evolve with scale, μ, are

$$\frac{d}{dt}\alpha_i^{-1} = -\frac{b_i}{2\pi} \ , \tag{126}$$

with $t = \log \mu/\Lambda$. The coefficients, b_i, are determined by the charges of the fields. For $SU(N)$ groups they are,

$$b = \frac{11}{3}N - \frac{2}{3}\sum_{fermions} C(r_f) - \frac{1}{3}\sum_{bosons} C(r_b). \tag{127}$$

For a fundamental (adjoint) representation $C = 1/2$ (N) and for a representation with charge Q under a $U(1)$ group $C = Q^2$.

In a supersymmetric $SU(N_c)$ gauge theory with N_f pairs of fields $(Q,\ \overline{Q})$, transforming in the fundamental representation, Eq. (127) takes on the simple form,

$$b = 3N_c - N_f \ . \tag{128}$$

Thus in the MSSM the coefficients are $b_i = (b_1, b_2, b_3) = (-33/5, -1, 3)$ to be compared with those of the SM, $b_i = (-41/10, 19/6, 7)$.

Exercise: Solve the gauge coupling RGEs for both the SM and the MSSM, starting from the weak scale where the gauge couplings are known. How does unification compare for the two cases, both in terms of the scale of closest approach and in terms of the size of the triangle at this scale - a measure of the success of unification? (*cf* Fig. 2).

7.2. *Superpotential terms*

The renormalisation group equations for superpotential terms have a very interesting feature, they are all multiplicatively renormalised. Thus if a tree-level parameter, such as the μ term, is small quantum corrections will not make it large, even after SUSY breaking. This result follows from holomorphy[51] of the superpotential and can be proven using the spurion techniques of Sec. 4, it has also been proven diagrammatically.[52]

One treats the parameters in the superpotential as chiral superfields that acquire a scalar VEV. Allowing these spurions to transform restores some global symmetries. The charge assignments of the spurions and the requirement that these fields always appear holomorphicaly in the superpotential forbid the superpotential from being renormalised in perturbation theory. For more detailed discussion of this remarkable result see for example, Ref. 15,53,54. Although the superpotential will not be renormalised in perturbation theory, there can be non-perturbative corrections which are critical to the dynamics of SUSY breaking in many models.

Unlike the the superpotential, the Kähler potential will be renormalised in perturbation theory so the superfields will have a wavefunction renormalisation. As a result of the wave function renormaliation the physical couplings will be renormalised, even though the superpotential is not, but only by these wavefunctions. Consequently the running of the superpotential terms is entirely determined by the anomalous dimensions of the fields involved in the coupling. We illustrate this by writing down the one-loop equations for the top Yukawa and the μ term, for the complete set of RGEs (up to two loops) see for example Ref. 50,

$$\begin{aligned} \frac{dy_t}{dt} &= \frac{y_t}{16\pi^2}\left(6|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right) \\ \frac{d\mu}{dt} &= \frac{\mu}{16\pi^2}\left(3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2\right) \end{aligned} \tag{129}$$

7.3. *Gaugino masses*

The one-loop RGE for the gaugino masses are related to those for the gauge couplings

$$\frac{d}{dt}M_i = -\frac{1}{8\pi^2}b_i g_i^2 M_i \ , \tag{130}$$

which results in the interesting fact that at one loop,

$$\frac{d}{dt}\frac{M_i}{g_i^2} = 0 \ . \tag{131}$$

Thus, if at the GUT scale, all the gauginos have the same mass - as is often assumed in gravity mediation, or the gauginos masses are generated proportional to their gauge coupling squared - as in minimal gauge mediation, then there is a prediction for the gaugino mass spectrum. The so-called unified gaugino mass boundary condition results in a ratio of masses at the weak scale of,

$$M_1 : M_2 : M_3 \approx 1 : 2 : 7 \ . \tag{132}$$

Resulting in a gluino significantly heavier than the charginos and neutralinos.

7.4. *Soft parameters*

The SUSY breaking parameters have additive renormalisation. I consider here, as examples, the left-handed stop mass and the soft masses for the Higgs doublets,

$$\begin{aligned} \frac{dm_{\tilde{q}_3}^2}{dt} &= \frac{1}{16\pi^2}\left(X_t + X_b - \frac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15}g_1^2|M_1|^2\right) \\ \frac{dm_{H_u}^2}{dt} &= \frac{1}{16\pi^2}\left(3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2\right) \\ \frac{dm_{H_d}^2}{dt} &= \frac{1}{16\pi^2}\left(3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2\right) \ . \end{aligned} \tag{133}$$

I have introduced $X_t = 2|Y_t|^2(m^2_{H_u} + m^2_{\tilde{q}_3} + m^2_{\tilde{u}^c_3}) + 2|a_t|^2$, $X_b = 2|Y_b|^2(m^2_{H_d} + m^2_{\tilde{q}_3} + m^2_{\tilde{d}^c_3}) + 2|a_b|^2$ and $X_\tau = 2|Y_\tau|^2(m^2_{H_d} + m^2_{\tilde{l}_3} + m^2_{\tilde{e}^c_3}) + 2|a_\tau|^2$. Here, and above, I have made the standard assumption that the Yukawa matrices are well approximated by setting all except the $(3,3)$ entries to zero. The equivalent RGEs for the first two generations only involve the gauge contributions. Furthermore the complete set of RGEs is substantially longer, I have just shown a few representative examples and provided references for the full set.

Although solving the RGEs has to be done numerically we can see a few interesting properties without resorting to numerics[j]. The gauge interactions push the soft scalar masses up as the RGEs run into the infrared (IR). This evolution is largest for the coloured particles and smallest for the right-handed sleptons. On the other hand, Yukawa interactions tend to drive scalar masses down in the IR, with the result that the third generation right-handed fields, $\tilde{t}^c$, $\tilde{b}^c$, $\tilde{\tau}^c$, will be lighter than their cousins from the first two generations, assuming flavour blind boundary conditions in the UV. The down-type Higgs has the same quantum numbers as the lepton doublets and so runs in a similar fashion, although at large $\tan\beta$ the corrections from the bottom Yukawa can have some effect. The up-type Higgs on the other hand experiences the effect of the large top Yukawa, without any compensating effect from the gluino, driving it mass down. It is possible that with unified boundary conditions for the scalar masses that $m^2_{h_u}$ is driven negative, while the other scalar masses remain positive. This is actually a very positive feature since it helps satisfy the conditions required for EWSB; Eq. (101) and Eq. (102). In this case the source of the W and Z mass comes from the renormalisation group evolution from the high scale and is known as radiative electroweak symmetry breaking.

As an example of the running of SUSY parameters and of radiative electroweak symmetry breaking I show in Fig. 7 the evolution of parameters for a standard MSUGRA benchmark point[59] called SPS1a. At the GUT scale the input parameters are,

$$m_0 = 100 \text{ GeV},\ m_{1/2} = 250 \text{ GeV},\ a_0 = 100 \text{ GeV},\ \tan\beta = 10,\ \mu > 0, \tag{134}$$

the remaining Higgs mass parameters are determined by the requirement of correct electroweak symmetry breaking, Sec. 5. The general features discussed above can be seen in the plot: the gluino is heavier than the wino

[j]There are many packages to solve the RGEs of the MSSM, *e.g.* Softsusy,[55] SuSpect,[56] SPheno[57] and ISASUSY.[58]

is heavier than the bino in the IR even though they are degenerate at the GUT scale. The squarks are heavier than the sleptons, and the up-type Higgs soft mass is pulled negative by the large top Yukawa, leading to radiative electroweak symmetry breaking. The full low energy spectrum for this point is shown in Fig. 8.

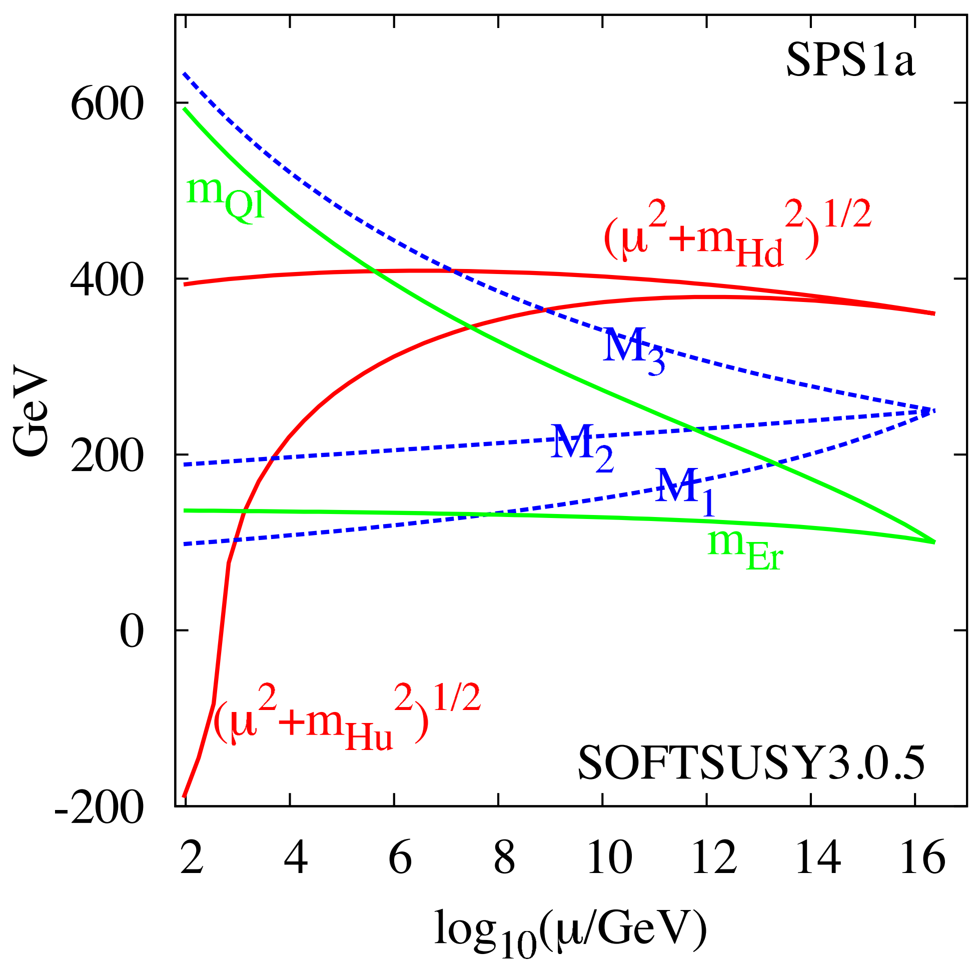

Fig. 7. Renormalisation group evolution of parameters for benchmark point SPS1a.[55]

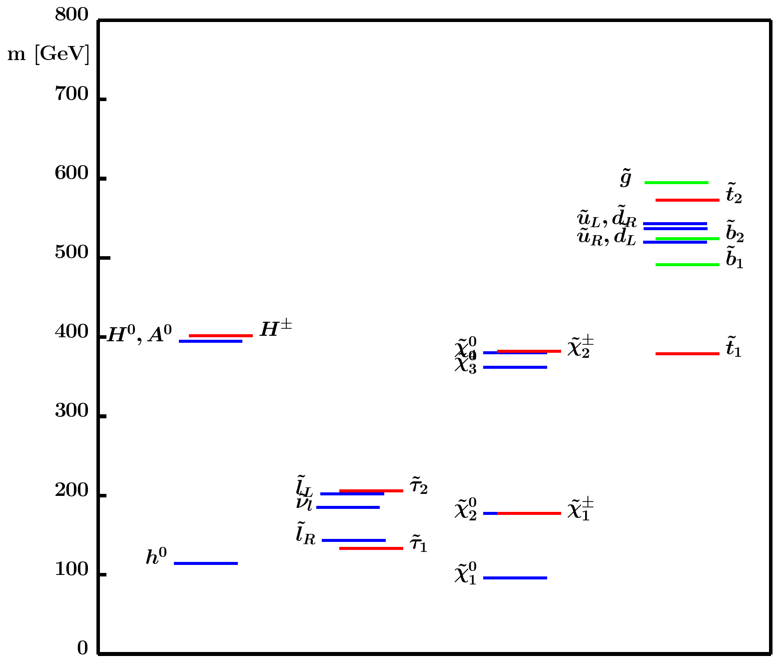

Fig. 8. Superpartner particle spectrum for benchmark point SPS1a,[59] see text for details.

7.5. *Effects of the hidden sector*

So far I have been implicitly assuming that the only light fields below the messenger scale are the fields of the MSSM, in this case the RGEs of the MSSM parameters are as described above. However, it is also possible that the messengers are not the lightest fields in the hidden sector. If the hidden sector is strongly coupled and contains light fields then it can have an appreciable affect on the running of the MSSM parameters.[60–62] In particular if the hidden sector is close to a conformal fixed point then its fields can acquire large anomalous dimensions, and the running from the mediation scale down to the scale at which conformal symmetry is broken can exponentially suppress certain operators involving hidden sector fields. If the hidden sector is weakly coupled these effects are small and can safely be ignored.

Given our inability to calculate anomalous dimensions in strongly coupled theories it is no longer possible to extrapolate from a weak scale observation of an MSSM parameter up to high scales and learn about UV physics. However, even in the presence of a strongly coupled hidden sector there are certain relations amongst MSSM parameters that remain and provide a limited probe of the mechanism of SUSY breaking. In certain classes of hidden sectors the running may be such that dangerous flavour changing operators, such as the off-diagonal scalar masses of Eq. (79) will be driven to zero even if they are $\mathcal{O}(1)$ at the mediation scale[63,64] or it may offer a solution to the μ-b problem.[62,65] So, although some predictivity has been lost with the inclusion of hidden sector running there are some potentially beneficial effects, and it should be emphasised that the hidden sector may well be of a type that cannot be ignored in the RGEs.

8. Supersymmetric Dark Matter

As mentioned in Sec. 3 the existence of relevant baryon- and lepton-number violating operators in the MSSM superpotential necessitates the introduction of R-parity[k]. This approach has the happy byproduct of making the lightest superpartner absolutely stable, and in a large fraction of parameter space the LSP has the correct properties to be the cosmological DM, which we now discuss. For a review of the evidence of, and other potential candidates for, DM see Neal Weiner's lectures notes, Ref. 3, in this volume.

The most studied DM candidate is a neutral weakly interacting massive particle (WIMP), the states in the MSSM that have the right properties are the lightest neutralino, the lightest snuetrino, and the gravitino. We will concentrate for the most part on the lightest neutralino, χ_1^0, but whatever the candidate I will refer to it from now on as χ.

In the fiery early moments of our universe all particles of the MSSM were in thermodynamic equilibrium, and very abundant. But as the universe cools, with the temperature T eventually dropping below the LSP's mass, the rate of annihilation of χ wins out against that of creation and the equilibrium abundance of χ becomes suppressed,

$$n_\chi = g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-m_\chi/T} \ , \tag{135}$$

here g_χ counts the number of internal degrees of freedom of the LSP. At the same time the universe is expanding, and at some point the expansion rate of the universe will exceed the annihilation rate of the DM, resulting in DM

[k]Actually, there are alternative approaches[9,10] that allow R-parity to be broken without dangerous rates of proton decay, leading to an unstable LSP, I will not discuss them further here.

"freezing out" and the number density (per co-moving volume) becoming fixed.

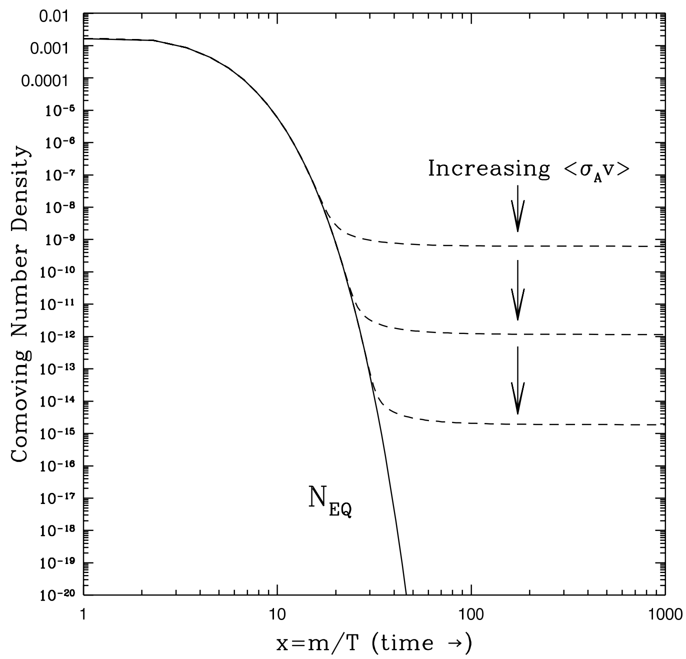

Fig. 9. The evolution of the DM (comoving) number density as the universe cools.

To understand this evolution in detail one must solve the Boltzmann equation in an expanding universe,

$$\frac{dn}{dt} + 3Hn = \langle \sigma v_{rel} \rangle (n_{eq}^2 - n^2) \tag{136}$$

where n_{eq} is the equilibrium number density, given by Eq. (135) and $H = \dot{a}/a$ is the expansion rate of the universe, a is the scale factor of the Friedmann-Robertson-Walker metric, see Michael Turner's lectures in this volume for more details Ref. 66. During the radiation dominated epoch when freeze out occurs the entropy density scales as $s \sim T^3$ and $H \sim T^2$. The Boltzmann equation is most easily solved in terms of the quantities $Y = n/s$ and $x = m_\chi/T$, the solution is show in Fig. 9.

Freeze out happens when the expansion term is comparable to the annihilation term, and for typical neutralino annihilation cross sections this occurs at $x \sim 20 - 25$, with only logarithmic sensitivity to the DM mass.

Furthermore this insensitivity of the freeze out temperature means that the present day DM abundance is determined almost entirely in terms of its annihilation cross section,

$$\Omega_{DM} \sim \frac{1}{\langle \sigma v_{rel} \rangle} \ . \tag{137}$$

The most recent observation of the DM abundance, from WMAP and other experiments,[67,68] gives $\Omega_{DM} = 0.213$. A detailed derivation of Eq. (137) reveals that a weak scale cross section, $\langle \sigma v \rangle \sim \alpha^2/M_W^2 \sim 1$ pb gives approximately the observed value for the DM abundance. This is often referred to as the "WIMP miracle", a particle with weak scale mass and annihilation cross section gives the correct relic abundance and the MSSM contains just such particles.

In the MSSM, and other models of BSM physics, it may not be sufficient to consider just the evolution of the DM particle in isolation. There are situations[69] that occur in sizeable regions of parameter space that require more detailed analysis:

Coannihilation If there is another MSSM state with mass within a few percent of the DM mass then its abundance at freeze out will not be negligible. For larger mass splittings the Maxwell-Boltzmann suppression Eq. (135) of its thermal abundance is large. The additional state(s) can take part in annihilation and in the determination of the relic abundance. Such a situation is referred to as "coannihilation" and can occur in the MSSM where the DM bino coannihilates with a nearly degenerate stau.

s-channel pole The annihilation cross section can be greatly altered if there is a state whose mass is close to twice the DM mass. If there are the necessary couplings DM may now annihilate through an s-channel resonance, leading to a greatly enhanced cross section. This can occur in the MSSM for a neutralino that is an admixture of gaugino and higgsino annihilating through the A^0 pole.

Solving for the DM abundance in the MSSM is a complicated business due to the complexity of the spectrum and couplings and the number of diagrams that contribute to the annihilation. Thankfully computer codes such as DarkSUSY[70] and Micromegas[71] exist that can numerically solve Eq. (136) taking into account both coannihilation and s-channel poles.

Notice that freeze out occurs when the neutralino is non-relativistic which means its annihilation should be dominated by S-wave. In addition, the neutralino is Majorana, and so its own antiparticle, and consequently the annihilation must take place in the antisymmetric spin 0 state. Thus annihilation to SM fermions is helicity suppressed, meaning that the annihilation cross section to light SM fermions is suppressed by m_f^2/m_χ^2. This rate is too small and the resulting relic abundance is too large! The LSP must be able to annihilate into top or vector bosons, the later requiring the LSP not be pure bino but contain an admixture of wino or higgsino, or must have coannihilations or resonances available to it to increase the annihilation cross section. In the CMSSM the regions of parameter space with the correct relic abundance can be identified with each of these cases[l].

The properties of DM can also be probed through direct (recoils of DM off "stuff" in the lab) and indirect (observation of the final states of "present-day" DM annihilations) searches. There are strong bounds on the scattering cross section of DM off nulcei[72] that for a weak scale particle like a neutralino mean that the LSP's coupling to the Z cannot be too large, ruling out a snuetrino as the DM. The situation of a gravitino LSP suffers from the converse problem, the coupling is far too small to ever allow detection of the DM in the lab. For a discussion of some of the phenomenology of a gravitino LSP see Ref. 73.

The details of DM direct and indirect detection are covered in far more detail in various reviews, for a review with special emphasis on supersymmetric DM see, for example, Ref. 74.

9. Onward

From a certain point onward there is no longer any turning back.
That is the point that must be reached.
–Franz Kafka

Although these lecture notes have only scratched the surface of what is a vast subject it is my hope that they contain enough information (and motivation!) for you to set off on your own into the supersymmetric world. There are many textbooks and review articles out there to assist you.

The definitive introduction to most aspects of this subject is the "Primer" by Stephen Martin.[75] For the collider phenomenology of supersymmetric theories see the TASI lecture notes of Maxim Perelstein[76] in this volume, or from previous years,[77] or the text book by Baer and Tata.[78]

[l]Although obtaining the correct relic abundance is often a constraint placed on scans of SUSY parameter space, more conservatively one can require that the relic LSP abundance should not be *larger* than the WMAP measurement. A smaller abundance (larger annihilation cross section) would then require an additional DM component *e.g.* an axion.

There have been many previous TASI lectures on various aspects of SUSY, see Refs. 15,54,79,80. On the more formal side of things the text book by Wess and Bagger[81] is an excellent resource.

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